CS 130 Homework 1

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The following problems are taken from exercises at the end of Section 1.1 of Gersting, 6e.

1 (Exercise 2) Given the truth values A true, B false, and C true, what is the truth value of each of the following WFFs?

a. $A \wedge (B \vee C)$

b. $(A \wedge B) \vee C$

c. $(A \wedge B)' \vee C$

d. $A' \vee (B' \wedge C)'$

2 (Exercise 5) Several forms of negation are given for each of the following statements. Which are correct?

a. The answer is either 2 or 3.

   1. Neither 2 nor 3 is the answer.
   2. The answer is not 2 or not 3.
   3. The answer is not 2 and it is not 3.

b. Cucumbers are green and seedy.

   1. Cucumbers are not green and not seedy.
   2. Cucumbers are not green or not seedy.
   3. Cucumbers are green and not seedy.

c. $2 < 7$ and 3 is odd.

   1. $2 > 7$ and 3 is even.
   2. $2 \geq 7$ and 3 is even.
   3. $2 \geq 7$ or 3 is odd.
   4. $2 \geq 7$ or 3 is even.

3 (Exercise 9) Let A, B, and C be the following statements:

$$\begin{aligned} A &= \text{Roses are red.} \\ B &= \text{Violets are blue.} \\ C &= \text{Sugar is sweet.} \end{aligned}$$

Translate the following compound statements into symbolic notation.

a. Roses are red and violets are blue.

b. Roses are red, and either violets are blue or sugar is sweet.

c. Whenever violets are blue, roses are red and sugar is sweet.

d. Roses are red only if violets aren't blue or sugar is sour.

e. Roses are red and, if sugar is sour, then either violets aren't blue or sugar is sweet.

4 (Exercise 17) Construct truth tables for the following WFFs. Note any tautologies or contradictions.

a. $(A \to B) \leftrightarrow A' \vee B$

b. $(A \wedge B) \vee C \to A \wedge (B \vee C)$

c. $A \wedge (A' \vee B')'$

d. $A \wedge B \to A'$

e. $(A \to B) \to ((A \vee C) \to (B \vee C))$

f. $A \to (B \to A)$

g. $A \wedge B \leftrightarrow B' \vee A'$

h. $(A \vee B') \wedge (A \wedge B)'$

i. $((A \vee B) \wedge C') \to A' \vee C$

5 (Exercise 27) Rewrite the following statement form with a simplified conditional expression, where the function `odd(n)` returns true if `n` is odd.

```
if (not ((value1 < value2) or odd(number)) or
    (not (value1 < value2) and odd(number))):
    # statement 1
else:
    #statement 2
```

6 (Exercise 32) Every compound statement is equivalent to a statement using only the connectives of conjunction and negation. To see this, we need to find equivalent WFFs for $A \vee B$ and for $A \to B$ using only $\wedge$ and $'$. These new statements can replace, respectively, any occurrences of $A \vee B$ and $A \to B$. (The connective $\leftrightarrow$ was defined in terms of other connectives, so we already know that it can be replaced by a statement using these other connectives.)

a. Show that $A \vee B$ is equivalent to $(A' \wedge B')'$.

b. Show that $A \to B$ is equivalent to $(A \wedge B')'$.

7 (Exercise 36) The binary connective $\downarrow$ is defined by the following truth table:

A	B	A ↓ B
F	F	T
F	T	F
T	F	F
T	T	F

Show that every compound statment is equivalent to a statement using only the connective $\downarrow$. (*Hint:* Use 6 and find equivalent statements for $A \wedge B$ and A' in terms of $\downarrow$.)