CS 130 Homework 2

Alex Vondrak

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1. Though the axiomatic system from page 7 of the notes is sound and complete, having fewer inference rules to choose from can make proofs longer. But, because it is sound and complete, we can build up the inference rules of natural deduction from page 6 of the notes. Using the proof system from page 7, prove the following inference rules from page 6.

   a. $Q' \wedge (P \to Q) \to P'$ — mt
   b. $(P \to Q) \wedge (Q \to R) \to (P \to R)$ — hs
   c. $(P \vee Q) \wedge P' \to Q$ — ds
   d. $P \to (P \vee Q)$ — add
      Hint: use Axiom 1 along with the hypothesis.

   Notice that we already use Modus Ponens and Conjunction; the symmetric cases of Disjunctive Syllogism and Addition are proved similarly to the above; and Simplification follows directly from the hypotheses. We skip the proof of Dilemma, as it is trickier.

   Upon completing this problem, you are free to use any of the inference rules from page 6 for proofs in propositional logic, knowing that they are justified.

The following problems are taken from exercises at the end of Section 1.2 of Gersting, 6e.

2. (Exercise 9) Justify each step in the proof sequence of

$$A \wedge (B \to C) \to (B \to (A \wedge C))$$

   1. A
   2. $B \to C$
   3. B
   4. C
   5. $A \wedge C$

3 (Exercise 11) Justify each step in the proof sequence of

$$A' \wedge B \wedge (B \to (A \vee C)) \to C$$

1. A'
2. B
3. $B \to (A \vee C)$
4. $A \vee C$
5. $(A')' \vee C$
6. $A' \to C$
7. C

4 (Exercise 17) Use propositional logic to prove that the argument is valid.

$$(A' \to B') \wedge B \wedge (A \to C) \to C$$

5 (Exercise 19) Use propositional logic to prove that the argument is valid.

$$(A \to (B \to C)) \to (B \to (A \to C))$$

6 (Exercise 21) Use propositional logic to prove that the argument is valid.

$$(A \to C) \wedge (C \to B') \wedge B \to A'$$