CS 130 Homework 3

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The following problems are taken from exercises at the end of Sections 1.3 and 1.4 of Gersting, 6e.

1 (Section 1.3, Exercise 2) What is the truth value of each of the following WFFs in the interpretation where the domain consists of the integers?

e. $(\forall x)(\forall y)[x < y \vee y < x]$
f. $(\forall x)\big[x < 0 \to (\exists y)[y > 0 \wedge x + y = 0]\big]$
g. $(\exists x)(\exists y)[x^2 = y]$
h. $(\forall x)[x^2 > 0]$

2 (Section 1.3, Exercise 4) Give the truth value of each of the following WFFs in the interpretation where the domain consists of people: $M(x, y)$ is "x is the mother of y", $F(x)$ is "x is female", $M(x)$ is "x is male".

a. $(\forall x)(\exists y)[M(y, x)]$
b. $(\exists x)(\forall y)[M(x, y)]$
c. $(\forall x)(\forall y)[M(x, y) \to M(y)]$
d. $(\exists x)(\exists y)[M(x, y) \wedge M(y)]$
e. $(\exists x)(\forall y)[M(x, y) \wedge F(y)]$

3 (Section 1.3, Exercise 5) For each WFF, find an interpretation in which it is true and one in which it is false.

a. $(\forall x)\big[(A(x) \vee B(x)) \wedge (A(x) \wedge B(x))'\big]$
b. $(\forall x)(\forall y)[P(x, y) \to P(y, x)]$
c. $(\forall x)[P(x) \to (\exists y)Q(x, y)]$
d. $(\exists x)[A(x) \wedge (\forall y)B(x, y)]$
e. $\big((\forall x)A(x) \to (\forall x)B(x)\big) \to (\forall x)[A(x) \to B(x)]$

4 (Section 1.4, Exercise 6) Justify each step in the following proof sequence of

$$(\exists x)[P(x) \to Q(x)] \to \big((\forall x)P(x) \to (\exists x)Q(x)\big)$$

1. $(\exists x)[P(x) \to Q(x)]$
2. $P(a) \to Q(a)$
3. $(\forall x)P(x)$
4. $P(a)$
5. $Q(a)$
6. $(\exists x)Q(x)$

5 (Section 1.4, Exercise 11) Prove that the WFF is a valid argument.

$$(\forall x)P(x) \wedge (\exists x)Q(x) \to (\exists x)[P(x) \wedge Q(x)]$$

6 (Section 1.4, Exercise 12) Prove that the WFF is a valid argument.

$$(\exists x)(\exists y)P(x, y) \to (\exists y)(\exists x)P(x, y)$$

7 (Section 1.4, Exercise 14) Prove that the WFF is a valid argument.

$$(\forall x)P(x) \wedge (\exists x)[P(x)]' \to (\exists x)Q(x)$$