

# CS 130 Homework 4

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The following problems are taken from exercises at the end of Sections 2.1 and 2.2 of Gersting, 6e.

- 1 (Section 2.1, Exercise 4) Provide counterexamples to the following statements:
- The number  $n$  is an odd integer if and only if  $3n + 5$  is an even integer.
  - The number  $n$  is an even integer if and only if  $3n + 2$  is an even integer.

- 2 (Section 2.1, Exercise 12) Give a direct proof of the statement:

The sum of even integers is even.

- 3 (Section 2.1, Exercise 13) Give a proof by contradiction of the statement:

The sum of even integers is even.

- 4 (Section 2.1, Exercise 21) Give a proof by contraposition of the statement:

If a number  $x$  is positive, so is  $x + 1$ .

- 5 (Section 2.1, Exercise 50) Prove or disprove the following statement:

The sum of any three consecutive integers is even.

- 6 (Section 2.1, Exercise 59) Prove or disprove the following statement:

For every positive integer  $n$ ,  $2^n + 1$  is prime.

- 7 (Section 2.2., Exercise 7) Prove that for  $n \geq 1$ , the following is true:

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

- 8 (Section 2.2., Exercise 51) Prove that for  $n \geq 1$ ,  $n^3 - n$  is divisible by 3.