CS 130 Homework 6

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The following problems are taken from exercises at the end of Section 3.1 and Section 4.1 of Gersting, 6e.

1 (Section 3.1, Exercise 13) Let

$$A = \{a,\ \{a\},\ \{\{a\}\}\} \qquad B = \{a\} \qquad C = \{\varnothing,\ \{a, \{a\}\}\}$$

Which of the following are true? For those that are not, where do they fail?

a. $B \subseteq A$
b. $B \in A$
c. $C \subseteq A$
d. $\varnothing \subseteq C$
e. $\varnothing \in C$
f. $\{a, \{a\}\} \in A$
g. $\{a, \{a\}\} \subseteq A$
h. $B \subseteq C$
i. $\{\{a\}\} \subseteq A$

2 (Section 3.1, Exercise 24) Find $\wp(S)$ for $S = \{\varnothing\}$.

3 (Section 3.1, Exercise 40) Let

$$A = \{a,\ \{a\},\ \{\{a\}\}\}$$
$$B = \{\varnothing,\ \{a\},\ \{a, \{a\}\}\}$$
$$C = \{a\}$$

be subsets of $U = \{\varnothing,\ a,\ \{a\},\ \{\{a\}\},\ \{a, \{a\}\}\}$. Find

a. $A \cap C$
b. $B \cap C'$
c. $A \cup B$
d. $\varnothing \cap B$
e. $(B \cup C) \cap A$
f. $A' \cap B$
g. $\{\varnothing\} \cap B$

4 (Section 3.1, Exercise 53) Prove that

$$(A \cap B) \subseteq A$$

where A and B are arbitrary sets.

5 (Section 4.1, Exercise 10) Let $S = \{0, 1, 2, 4, 6\}$. Test the following binary relations on S for reflexivity, symmetry, antisymmetry, and transitivity.

a. $\rho = \{(0,0),\ (1,1),\ (2,2),\ (4,4),\ (6,6),\ (0,1),\ (1,2),\ (2,4),\ (4,6)\}$

b. $\rho = \{(0,1),\ (1,0),\ (2,4),\ (4,2),\ (4,6),\ (6,4)\}$

c. $\rho = \{(0,1),\ (1,2),\ (0,2),\ (2,0),\ (2,1),\ (1,0),\ (0,0),\ (1,1),\ (2,2)\}$

d. $\rho = \{(0,0),\ (1,1),\ (2,2),\ (4,4),\ (6,6),\ (4,6),\ (6,4)\}$

e. $\rho = \varnothing$

6 (Section 4.1, Exercise 19) Two additional properties of a binary relation ρ are defined as follows:

$$\rho \text{ is } \textit{irreflexive} \text{ means } (\forall x \in S)[(x, x) \notin \rho]$$
$$\rho \text{ is } \textit{asymmetric} \text{ means } (\forall x, y \in S)[(x, y) \in \rho \rightarrow (y, x) \notin \rho]$$

a. Give an example of a binary relation ρ on set $S = \{1, 2, 3\}$ that is neither reflexive nor irreflexive.

b. Give an example of a binary relation ρ on set $S = \{1, 2, 3\}$ that is neither symmetric nor asymmetric.

c. Prove that if ρ is an asymmetric relation on a set S, then ρ is irreflexive.

d. Prove that if ρ is an irreflexive and transitive relation on a set S, then ρ is asymmetric.

e. Prove that if ρ is a nonempty, symmetric, and transitive relation on a set S, then ρ is not irreflexive.