# CS 130 Midterm Exam

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1 a. Give the fully-parenthesized propositional WFF that is identical to the following propositional WFF (by adding an explicit pair of parentheses for each operator):

$$A \wedge B \vee C' \rightarrow D \leftrightarrow C' \wedge B \vee A$$

b. Give the propositional WFF without unnecessary parentheses that is identical to the following propositional WFF (by removing all unnecessary parentheses):

$$\Big((A \vee B) \wedge \big(((B \rightarrow C)') \rightarrow C\big)\Big)$$

2 Which one of the following is an instance of Axiom 1 from page 7 of the notes?

a. $(((A \leftrightarrow C) \rightarrow (B \wedge A)) \wedge C) \rightarrow (C \rightarrow (((A \leftrightarrow C) \rightarrow (B \wedge A)) \wedge C)')$

b. $((((D \vee C) \leftrightarrow (A \leftrightarrow B)) \vee B') \rightarrow ((B' \vee A) \wedge (B \leftrightarrow B)')) \rightarrow (((D \vee C) \leftrightarrow (A \leftrightarrow B)) \vee B')$

c. $((D \wedge C)' \rightarrow (C' \wedge A)) \rightarrow (((D \rightarrow D) \rightarrow (C \vee A))' \rightarrow ((D \wedge C)' \rightarrow (C' \wedge A)))$

d. $D' \rightarrow (D' \rightarrow (D \leftrightarrow ((B \wedge D) \vee (C \wedge B))))$

3 What's wrong with the following "proof" of $(\exists x)[P(x)] \wedge (\exists x)[Q(x)] \rightarrow (\exists x)[P(x) \wedge Q(x)]$ (which is an invalid sentence)?

| | |
|---|---|
| 1. $(\exists x)[P(x)]$ | hyp |
| 2. $P(x)$ | 1, ei |
| 3. $(\exists x)[Q(x)]$ | hyp |
| 4. $Q(x)$ | 3, ei |
| 5. $P(x) \wedge Q(x)$ | 2, 4, conj |
| 6. $(\exists x)[P(x) \wedge Q(x)]$ | 5, eg |

4 a. Prove that $(C \rightarrow A)' \rightarrow (A \rightarrow B) \wedge C$ is a tautology by using a truth table.

b. Prove that $(C \rightarrow A)' \rightarrow (A \rightarrow B) \wedge C$ is a tautology by using propositional logic.

5 Prove that the following sentence is valid by using predicate logic.

$$\big((\exists x)[P(x) \wedge Q(x)']\big)' \rightarrow (\forall x)[P(x) \rightarrow Q(x)]$$

*See other side*

6 Prove that the following propositional WFF is a tautology by using propositional logic.

$$(\mathsf{A} \to \mathsf{B}) \wedge (\mathsf{B} \to (\mathsf{C} \wedge \mathsf{D})') \wedge (\mathsf{C}' \to \mathsf{E}) \to ((\mathsf{D}' \vee \mathsf{E})' \to \mathsf{A}')$$

7 Consider the following truth table, which defines the ☺ operator:

| A | B | A ☺ B |
|---|---|---|
| F | F | F |
| F | T | T |
| T | F | F |
| T | T | F |

Find a propositional WFF (i.e., one using the operators $\wedge$, $\vee$, $\leftrightarrow$, $\to$, and/or $'$) that is equivalent to A ☺ B.

*See other side*