

Solutions to Practice Proofs

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Try to do the practice proofs on your own, then consult these solutions to check your answers. The following are my solutions—other valid solutions may still be slightly different.

$$\boxed{1} \quad (A \rightarrow B') \wedge (C \rightarrow B) \wedge ((C' \vee D) \rightarrow E) \wedge (E \rightarrow G) \rightarrow (A \rightarrow G)$$

1. $A \rightarrow B'$	hyp
2. $C \rightarrow B$	hyp
3. $(C' \vee D) \rightarrow E$	hyp
4. $E \rightarrow G$	hyp
5. A	hyp
6. B'	5, 1, mp
7. C'	6, 2, mt
8. $C' \vee D$	7, add
9. E	8, 3, mp
10. G	9, 4, mp

$$\boxed{2} \quad (A \rightarrow (B \wedge C)) \wedge ((B \vee D) \rightarrow E) \wedge (G' \vee (D \vee C')) \wedge (E \leftrightarrow G) \rightarrow (A \rightarrow D)$$

1. $A \rightarrow (B \wedge C)$	hyp
2. $(B \vee D) \rightarrow E$	hyp
3. $G' \vee (D \vee C')$	hyp
4. $E \leftrightarrow G$	hyp
5. A	hyp
6. $B \wedge C$	5, 1, mp
7. B	6, simp
8. C	6, simp
9. $B \vee D$	7, add
10. E	9, 2, mp
11. $(E \rightarrow G) \wedge (G \rightarrow E)$	4, equiv
12. $E \rightarrow G$	11, simp
13. G	10, 12, mp
14. $G \rightarrow (D \vee C')$	3, imp
15. $D \vee C'$	13, 14, mp
16. $(C')'$	8, dn
17. D	15, 16, ds

3 $((P' \vee Q') \rightarrow (A \vee B)) \wedge (P \rightarrow C) \wedge (B \rightarrow C) \rightarrow (C' \rightarrow A)$

1. $(P' \vee Q') \rightarrow (A \vee B)$

hyp

2. $P \rightarrow C$

hyp

3. $B \rightarrow C$

hyp

4. C'

hyp

5. P'

4, 2, mt

6. $P' \vee Q'$

5, add

7. $A \vee B$

6, 1, mp

8. B'

4, 3, mt

9. A

7, 8, ds

4 $((A \vee B) \rightarrow (C \vee D)) \wedge (C \rightarrow E) \wedge (A \wedge E') \rightarrow (D \vee W)$

1. $(A \vee B) \rightarrow (C \vee D)$	hyp
2. $C \rightarrow E$	hyp
3. $A \wedge E'$	hyp
4. A	3, simp
5. $A \vee B$	4, add
6. $C \vee D$	5, 1, mp
7. E'	3, simp
8. C'	7, 2, mt
9. D	8, 6, ds
10. $D \vee W$	9, add

5 $((A' \vee B') \rightarrow Q') \wedge (A' \rightarrow (P \rightarrow Q)) \wedge ((A \rightarrow D) \wedge D') \rightarrow P'$

1. $(A' \vee B') \rightarrow Q'$	hyp
2. $A' \rightarrow (P \rightarrow Q)$	hyp
3. $(A \rightarrow D) \wedge D'$	hyp
4. $A \rightarrow D$	3, simp
5. D'	3, simp
6. A'	5, 4, mt
7. $A' \vee B'$	6, add
8. Q'	7, 1, mp
9. $P \rightarrow Q$	6, 2, mp
10. P'	8, 9, mt

$$\boxed{6} \quad (A \rightarrow (B \rightarrow C)) \wedge (B' \rightarrow (Q \vee G)) \wedge ((G \wedge H') \rightarrow (D \rightarrow B)) \wedge ((A \wedge C') \vee H) \wedge (H' \wedge Q') \rightarrow D'$$

1. $A \rightarrow (B \rightarrow C)$	hyp
2. $B' \rightarrow (Q \vee G)$	hyp
3. $(G \wedge H') \rightarrow (D \rightarrow B)$	hyp
4. $(A \wedge C') \vee H$	hyp
5. $H' \wedge Q'$	hyp
6. H'	5, simp
7. $A \wedge C'$	4, 6, ds
8. A	7, simp
9. $B \rightarrow C$	8, 1, mp
10. C'	7, simp
11. B'	10, 9, mt
12. $Q \vee G$	11, 2, mp
13. Q'	5, simp
14. G	13, 12, ds
15. $G \wedge H'$	14, 6, conj
16. $D \rightarrow B$	15, 3, mp
17. D'	11, 16, mt

$$\boxed{7} \quad ((A \vee B) \rightarrow (C \vee D)) \wedge (C \rightarrow E) \wedge (C \vee P') \wedge (A \wedge E') \wedge (P \vee (D \rightarrow Z)) \rightarrow Z$$

1. $(A \vee B) \rightarrow (C \vee D)$	hyp
2. $C \rightarrow E$	hyp
3. $C \vee P'$	hyp
4. $A \wedge E'$	hyp
5. $P \vee (D \rightarrow Z)$	hyp
6. A	4, simp
7. $A \vee B$	6, add
8. $C \vee D$	7, 1, mp
9. E'	4, simp
10. C'	9, 2, mt
11. P'	10, 3, ds
12. $D \rightarrow Z$	5, 11, ds
13. D	8, 10, ds
14. Z	13, 12, mp

8 $((X \vee Y) \rightarrow (Y \vee Z)) \wedge (X \wedge (Y \rightarrow Y')) \wedge (Z \rightarrow Z') \rightarrow (Y' \vee Z')$

1. $(X \vee Y) \rightarrow (Y \vee Z)$

hyp

2. $X \wedge (Y \rightarrow Y')$

hyp

3. $Z \rightarrow Z'$

hyp

4. X

2, simp

5. $X \vee Y$

4, add

6. $Y \vee Z$

5, 1, mp

7. $Y \rightarrow Y'$

2, simp

8. $Y' \vee Z'$

6, 7, 3, dil

$$\boxed{9} \quad ((B' \vee C') \rightarrow (A \rightarrow W)) \wedge (A' \rightarrow (U \rightarrow Z')) \wedge (U \wedge X') \wedge (X \vee (W' \wedge B')) \wedge (Z' \rightarrow (S' \vee X)) \rightarrow S'$$

1. $(B' \vee C') \rightarrow (A \rightarrow W)$	hyp
2. $A' \rightarrow (U \rightarrow Z')$	hyp
3. $U \wedge X'$	hyp
4. $X \vee (W' \wedge B')$	hyp
5. $Z' \rightarrow (S' \vee X)$	hyp
6. X'	3, simp
7. $W' \wedge B'$	6, 4, ds
8. B'	7, simp
9. $B' \vee C'$	8, add
10. $A \rightarrow W$	9, 1, mp
11. W'	7, simp
12. A'	11, 10, mt
13. $U \rightarrow Z'$	12, 2, mp
14. U	3, simp
15. Z'	14, 13, mp
16. $S' \vee X$	15, 5, mp
17. S'	6, 16, ds

$$\boxed{10} \quad ((P' \vee Q') \rightarrow (R' \vee S')) \wedge (P \rightarrow U) \wedge (W' \rightarrow (U' \wedge Z')) \wedge ((S' \rightarrow Z) \wedge (X \wedge Y)') \wedge (W' \vee (X \wedge Y)) \rightarrow (R' \wedge W')$$

1. $(P' \vee Q') \rightarrow (R' \vee S')$	hyp
2. $P \rightarrow U$	hyp
3. $W' \rightarrow (U' \wedge Z')$	hyp
4. $(S' \rightarrow Z) \wedge (X \wedge Y)'$	hyp
5. $W' \vee (X \wedge Y)$	hyp
6. $(X \wedge Y)'$	4, simp
7. W'	5, 6, ds
8. $U' \wedge Z'$	3, 7, mp
9. U'	8, simp
10. P'	2, 9, mt
11. $P' \vee Q'$	10, add
12. $R' \vee S'$	1, 11, mp
13. Z'	8, simp
14. $S' \rightarrow Z$	4, simp
15. $(S')'$	13, 14, mt
16. R'	12, 15, ds
17. $R' \wedge W'$	7, 16, conj