CS 130 Homework 4

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Practice Problems

You are not required to turn these in.

Gersting, 6e: Section 1.2, Exercises 12–22 and Section 1.4, Exercises 10–27

Book of Proof: N/A (this book is light on formal logic)

Note that Gersting uses a different syntax than we've been using in class. Basically, they write $A \wedge B \wedge C \longrightarrow D$ to mean what we've written as $A, \quad B, \quad C \quad \vdash \quad D$. They're functionally equivalent (cf. the Deduction Theorem and Exportation), but may be hard to read at first—especially because the textbook also elides parentheses and square brackets a lot.

Turn-in Problems

Prove the following theorems using the predicate logic proof system. You may use any previously proven result from the homework/lecture and any theorem from the list provided at `http://www.csupomona.edu/~ajvondrak/cs/130/12/winter/theorems.pdf`.

1. $\vdash \quad (\forall x[\forall y[A(x,y)]] \implies \forall y[\forall x[A(y,x)]])$

2. (a) $\neg\exists x[A(x)] \quad \vdash \quad \forall x[\neg A(x)]$
   (b) $\forall x[\neg A(x)] \quad \vdash \quad \neg\exists x[A(x)]$
   (c) $\exists x[\neg A(x)] \quad \vdash \quad \neg\forall x[A(x)]$
   (d) $\neg\forall x[A(x)] \quad \vdash \quad \exists x[\neg A(x)]$

3. $\vdash \quad (\exists x[(A(x) \implies B(x))] \implies (\forall x[A(x)] \implies \exists x[B(x)]))$

4. $\vdash \quad \forall y[\exists x[(A(y,x) \implies A(y,y))]]$

Hints:

- For $\boxed{2}$(a)–(d), instead of writing down and/or looking for a $\exists$ in your formula, use the "expanded" form.
- For $\boxed{2}$(c), prove the lemma

$$\forall x[A(x)] \qquad \vdash \qquad \forall x[\neg\neg A(x)]$$

  then apply the Deduction Theorem to get the tautology

$$\vdash \qquad (\forall x[A(x)] \implies \forall x[\neg\neg A(x)])$$

  which you can use as a line in your proof.
- For $\boxed{2}$(d), prove the lemma

$$\forall x[\neg\neg A(x)] \qquad \vdash \qquad \forall x[A(x)]$$

  then apply the Deduction Theorem to get the tautology

$$\vdash \qquad (\forall x[\neg\neg A(x)] \implies \forall x[A(x)])$$

  which you can use as a line in your proof.
- In general, instead of proving something of the form $\vdash (A \implies B)$ directly, it's usually easier to prove $A \vdash B$ so you actually have a hypothesis. Then use the Deduction Theorem on this result to show $\vdash (A \implies B)$ must be true.