CS 130 Homework 5

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Due: February 10, 2012

Practice Problems

You are not required to turn these in.

Gersting, 6e: Section 2.1, Exercises 1–20

Book of Proof: Chapter 4, Exercises 1–10; Chapter 5, Exercises 1–10; Chapter 6, Exercises 1–10

Note that Book of Proof uses the notation "$x \in \mathbb{Z}$" to mean "x is an integer", "$x \in \mathbb{R}$" to mean "x is a real number", and "$x \mid y$" to mean "x divides evenly into y".

Turn-in Problems

We're not done with formal logic quite yet! After all, it's always present "behind the scenes" of our informal proofs. In this homework, we'll explore the special forms of proof studied in class: proof by contraposition, proof by contradiction, and proof by cases.

1. For proving certain implications, Proof by Contraposition might rely on an alternative inference rule:

$$(P \implies Q) \quad \vdash \quad (\neg Q \implies \neg P) \qquad \textbf{(Contraposition)}$$

   (a) Explain why this rule isn't quite the same as Axiom 3i from Homework 2.

   (b) Prove Contraposition using propositional logic.
   Hint: Theorems 6 & 7 from the Homework 2 Practice Problems will be useful. For reference, my proof is six lines long.

2. Proof by Contradiction can be formulated based on many similar but distinct propositional logic theorems, including

$$\vdash \quad (Q \implies (\neg Q \implies P)) \qquad \textbf{(Duns Scotus Law)}$$
$$\vdash \quad (\bot \implies P) \qquad \textbf{(Ex Falso Quodlibet)}$$
$$\vdash \quad ((\neg A \implies A) \implies A) \qquad \textbf{(Law of Clavius)}$$
$$(\neg P \implies \bot) \quad \vdash \quad P \qquad \textbf{(Reductio Ad Absurdum)}$$

where $\bot$ represents falsehood (with standard Boolean operators, typically $(Q \land \neg Q)$).

(a) Prove that the Law of Clavius is a tautology by writing a truth table.

(b) What is the truth value of the antecedent of the Law of Clavius when A is true? When A is false?

(c) In the Law of Clavius, A is the thing we're aiming to prove. By the converse of the Deduction Theorem, we know that

$$(\neg A \implies A) \qquad \vdash \qquad A$$

So, if we can show that $(\neg A \implies A)$ is true, we can conclude that A must also be true. In English, briefly explain a general strategy for proving $(\neg A \implies A)$ is true.

3 Proof by Cases relies on the following inference rule:

$$(A_1 \implies B), \quad (A_2 \implies B) \qquad \vdash \qquad ((A_1 \vee A_2) \implies B) \qquad \textbf{(Proof by Cases)}$$

(a) Give the justifications for the following formal proof of Proof by Cases.

1. $(A_1 \implies B)$
2. $(A_2 \implies B)$
3. $(\neg A_1 \implies A_2)$
4. $(\neg B \implies \neg A_1)$
5. $(\neg B \implies A_2)$
6. $(\neg B \implies B)$
7. $((\neg B \implies B) \implies B)$
8. B

(b) Is the proof in (a) more of a *Proof by Contraposition*, a *Proof by Contradiction*, or something else entirely? Explain.

(c) Using your answer to (b), translate the formal proof in (a) into an informal proof.