CS 130 Homework 6

Alex Vondrak

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Practice Problems

You are not required to turn these in.

Gersting, 6e: Section 2.2, Exercises 1–20; Section 2.4, Exercises 1–15

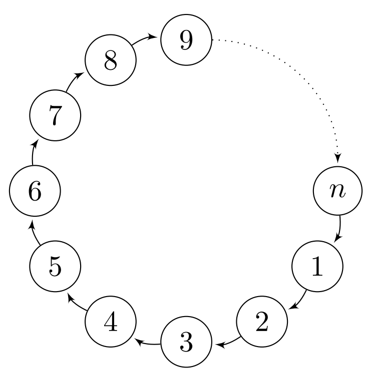
Book of Proof: Chapter 10, Exercises 1–15

Note that Book of Proof uses the notation "$x \in \mathbb{N}$" to mean "x is a natural number".

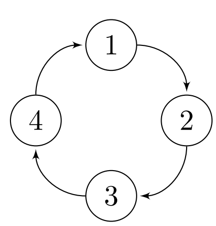
Turn-in Problems

For this homework, you'll be taking a look at some weird counting problems and figuring out what recursive relationships can be found.

Suppose we have a circle of n consecutive chairs.



Now, consider selecting some number of chairs from this circle. How many ways can we select a group of chairs that are *non*consecutive? Call this the *chair-circle number* on n chairs, and denote it C_n. For instance, in a circle of $n = 4$ chairs,

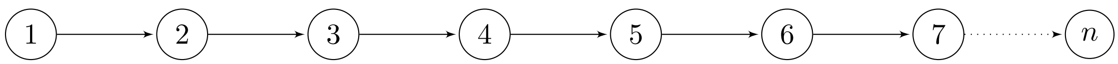


we can select the following groups of nonconsecutive chairs:

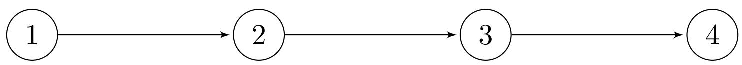
- No chairs at all!
- Just one chair (i.e., just chair 1, or just chair 2, or 3, or 4)
- Chairs 1 and 3
- Chairs 2 and 4

for a total of 7 possible groups. Thus, $C_4 = 7$.

Secondly, consider a sequence of n chairs in a line.



Suppose we select some group of chairs from this line. How many ways can we select a group of chairs that are *non*consecutive? Call this the *chair-line number* on n chairs, and denote it c_n. For instance, in a line of $n = 4$ chairs,



we can select the following groups of nonconsecutive chairs:

- No chairs at all!
- Just one chair (chair 1, 2, 3, or 4) on its own
- Chairs 1 and 3
- Chairs 1 and 4
- Chairs 2 and 4

for a total of 8 possible groups. Thus, $c_4 = 8$.

$\boxed{1}$ (a) What is the value of c_1? c_2? c_3? c_5?

Note: in a "line" of 1 chair, the chair is *not* considered consecutive with itself.

(b) Prove that $c_n = c_{n-1} + c_{n-2}$ for any $n \geq 3$.

Hint: Do a proof by cases. Either an n-chair group contains chair 1, or it does not contain chair 1. Add the respective number of possibilities together by the principle of addition.

$\boxed{2}$ (a) What is the value of C_1? C_2? C_3? C_5?

Note: in a "circle" of 1 chair, the chair is considered consecutive with itself.

(b) Prove that $C_n = c_{n-1} + c_{n-3}$ for any $n \geq 4$.

Hint: Same strategy as in $\boxed{1}$(b).

$\boxed{3}$ From the relationships in $\boxed{1}$ and $\boxed{2}$, prove that $C_n = C_{n-1} + C_{n-2}$ for any $n \geq 3$.