CS 130 Homework 8

Alex Vondrak

DUE: March 2, 2012

Practice Problems

You are not required to turn these in.

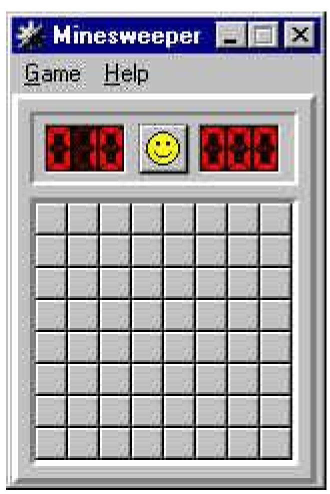
Gersting, 6e: Section 3.3, Exercises 17–23; Section 3.4, Exercises 1–19

Book of Proof: Section 3.3, Exercises 1–14; Section 3.5, Exercises 1–10; Section 12.3, Exercises 1–6

Turn-in Problems

In this homework, you'll be thinking about Minesweeper. In case you haven't used a computer since the 1980s, Minesweeper is a puzzle game that involves some configuration of spaces that you click on while trying to uncover where mines are. If you click on a space that hides a mine, you lose; if you click on a space that doesn't hide a mine, you uncover a number that tells you how many mines are adjacent to that space.

For those who suck at Minesweeper like I do, you'll be familiar with the classic beginner's configuration: an 8×8 board of squares across which there are 10 randomly-placed mines.



For example, the mines might be distributed like

	A	B	C	D	E	F	G	H
1		•			•			
2				•				
3		•			•			
4			•				•	
5								
6								
7					•			•
8	•							

$\boxed{1}$ (a) How many different initial configurations are possible on an 8×8 board with 10 mines? Explain.

(b) How many different initial configurations are possible on an $n \times n$ board with m mines? Explain.

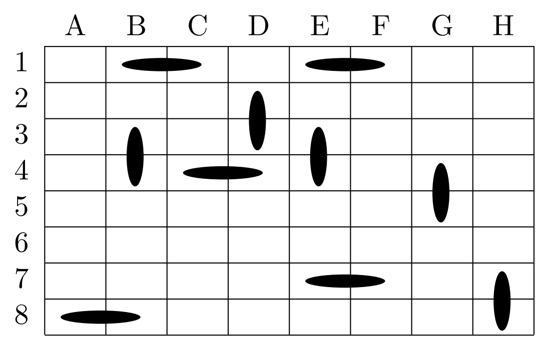
$\boxed{2}$ Suppose you're building a program to play Minesweeper for you, but you keep the program pretty dumb: it decides on a sequence of a moves ahead of time, then doesn't change its mind as it's given more information (like whether it's hit a mine or whatnot).

Let a *strategy* be the sequence of moves that the computer intends to play. The longest this sequence could be on the beginner board is $8 \times 8 = 64$ moves—one click for each square. For example, the row-by-row strategy would be all 64 moves in the order 1A, 1B, 1C, ..., 1H, 2A, 2B, 2C, ..., 2H, 3A, ..., 8H.

(a) How many possible 64-move strategies are there on the 8×8 board? Explain.

(b) How many possible m-move strategies are there on an $n \times n$ board? Explain.

$\boxed{3}$ Now suppose you want to make a new version of Minesweeper where you uncover *missiles* instead of mines. Unlike mines, which are placed in a single square, missiles are two squares long and can be placed either horizontally or vertically. No missiles can overlap or "fall off" the edge of the board. For example, the following is a possible configuration for 10 missiles.



I claim it is not possible to place missiles on an 8×8 board so that *every* square is covered by some missile *except* squares 1A and 8H (i.e., so that only squares 1A and 8H are "safe"). Prove it.

Hint: consider assigning the squares alternating colors, like a chess board. Then every missile covers two particular sorts of squares. By the Pigeonhole Principle, can there exist a 1-1 mapping between the sorts of squares?