Counting

CS 130

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Multiple Choice Question

Suppose you're ordering a dessert. There are 3 different types of ice cream and 4 different types of cake. How many different ways can you order a dessert?

(A) 12

(B) 7

(C) 2

(D) 1

Multiple Choice Question

Suppose you're buying a vehicle. The dealer has 23 different cars and 14 different trucks. How many choices do you have for your purchase?

(A) 23

(B) 14

(C) 37

(D) 322

Multiple Choice Question

In a fine-dining class, you must pick a wine to taste: either a red wine or a white wine. How many choices are there for your sample?

(A) # red wines + # white wines

(B) # red wines $\times$ # white wines

(C) Neither of the above

Multiple Choice Question

Let's say you're trying to pick one more class for your schedule. Your choice is between an elective (as per the CS curriculum sheet) or a math class. Assuming prerequisites aren't a problem, how many choices do you have for your class?

(A) # electives $+$ # math classes

(B) # electives $\times$ # math classes

(C) Neither of the above

The Addition Principle

Definition

Suppose E_1 and E_2 are two events, where

- E_1 has n_1 possible outcomes,
- E_2 has n_2 possible outcomes, and
- neither event affects the other (they are disjoint).

Then the total number of possible outcomes for the event "E_1 or E_2" is

$$n_1 + n_2$$

Multiple Choice Question

Let (pairwise) disjoint events E_1, E_2, E_3 have n_1, n_2, n_3 possible outcomes (respectively). How many possible outcomes are there for the event

$$\text{“}E_1 \text{ or } E_2 \text{ or } E_3\text{”}$$

?

(A) $(n_1 + n_2) \times n_3$

(B) $n_1 \times n_2 \times n_3$

(C) $(n_1 + n_2) + (n_2 + n_3) + (n_1 + n_3)$

(D) $n_1 + n_2 + n_3$

Result: A Generalization

Suppose the pairwise disjoint events $E_1, E_2, \ldots, E_m$ have the respective number of possible outcomes $n_1, n_2, \ldots, n_m$.
Then the event

$$\text{“}E_1 \text{ or } E_2 \text{ or } \ldots \text{or } E_m\text{”}$$

has

$$n_1 + n_2 + \cdots + n_m$$

possible outcomes.

Multiple Choice Question

If you own 3 shirts and 5 pairs of pants, how many different outfits (shirt and pants) could you make?

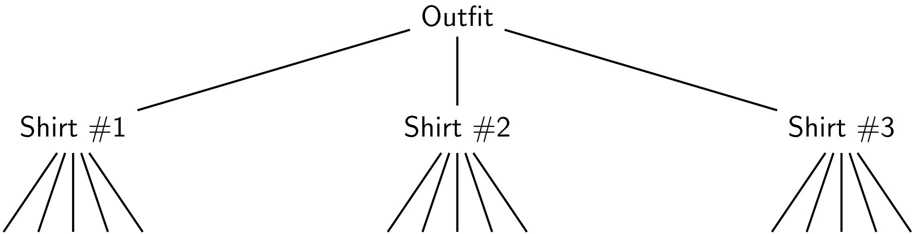
(A) 3×5

(B) $3 + 5$

(C) 3^5

(D) 5^3

Decision Trees



First you pick a shirt, then you pick a pair of pants:

Shirt #1: How many choices do you have for your pants?

OR Shirt #2: How many choices do you have for your pants?

OR Shirt #3: How many choices do you have for your pants?

The Multiplication Principle

Definition

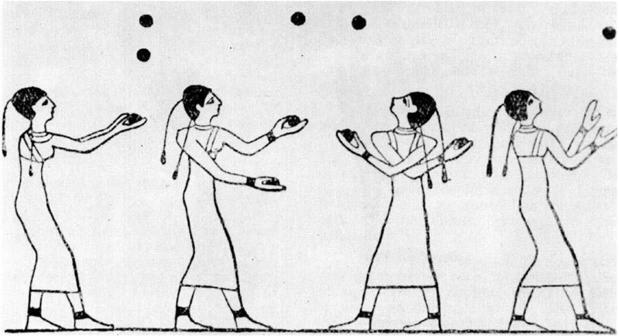
Suppose E_1 and E_2 are two events, where

- E_1 has n_1 possible outcomes and
- E_2 has n_2 possible outcomes.

Then the total number of possible outcomes for the event "E_1 and E_2" is

$$n_1 \times n_2$$

Juggling



Ancient Egyptian juggling (c. 1994–1781 B.C.)

`http://www.youtube.com/watch?v=sBHGzRxfeJY`
Claude Shannon juggling (c. 1985)

What Is Juggling?

- We have some number of balls, b
- We have some number of throws, n
  - In principle, a pattern may carry on forever
- Possible throws are largely governed by physics...
- ...Need to make simplifying assumptions
  - One ball lands at a time
  - One ball is thrown at a time
  - Make no distinction between "crazy" throws—just which ball is thrown

Queue Notation

- Only one ball in hand at a time (or so we can pretend)
- *How* we throw that ball dictates *when* it'll be thrown again
- Thus, can designate each throw by the "position" to which it's thrown

Example

Position 3	B		R		G		B
Position 2	G		B		R		G
Position 1	R	3	G	3	B	3	R

More Examples

Example

Position 3	B		R		R		B
Position 2	G		B	$\nearrow$ 2	G		R
Position 1	R	$\nearrow$ 3	G		B	$\nearrow$ 3	G

Example

Position 3	B		R		G		G
Position 2	G		B		R		R
Position 1	R	$\nearrow$ 3	G	$\nearrow$ 3	B	$\xrightarrow{1}$	B

Multiple Choice Question

How many balls are used in the following pattern?

Position 3	B		B		B
Position 2	G		G		G
Position 1	R	$\xrightarrow[1]{}$	R	$\xrightarrow[1]{}$	R

(A) 1

(B) 2

(C) 3

(D) Not enough information

Multiple Choice Question

How many patterns of length 1 exist using at most 3 balls?

(A) 1

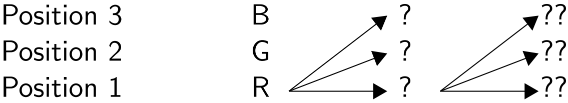
(B) 2

(C) 3

(D) ∞

Multiple Choice Question

How many patterns of length 2 exist using at most 3 balls?



(A) $3 + 3$

(B) 3×3

(C) 3^3

(D) Depends on what the first throw is

Multiple Choice Question

How many juggling patterns of length n exist using at most b balls?

(A) b^n

(B) $b \times b$

(C) $b \times n$

(D) $b + n$

Multiple Choice Question

Assume all phone numbers have the format

$$n_1 n_2 n_3 - n_4 n_5 n_6 n_7$$

where each n_i is a digit between 0 and 9. How many phone numbers exist?

(A) 7^9

(B) 9^7

(C) 7^{10}

(D) 10^7

Multiple Choice Question

Assume all license plates have the format

number letter letter letter number number number

where each letter is one of A–Z and each number is one of 0–9. How many license plates are possible?

(A) $10^4 \times 26^3$

(B) $10^4 + 26^3$

(C) $(10 + 26)^7$

(D) $(10 \times 26)^7$

Multiple Choice Question

Assume all phone numbers have the format

$$n_1 n_2 n_3 - n_4 n_5 n_6 n_7$$

where each n_i is a digit between 0 and 9. How many phone numbers exist that have no duplicated digits?

(A) 10^7

(B) 7^7

(C) $10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4$

(D) $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

Multiple Choice Question

If you own 12 shirts, 132 pairs of pants, and 10 dresses, how many different outfits (either shirt and pants *or* just a dress) could you make?

(A) $12 + 132 + 10$

(B) $12 + 132 \times 10$

(C) $12 \times 132 + 10$

(D) $12 \times 132 \times 10$

Multiple Choice Question

Suppose you flip a coin 4 times and write down the results in order: H for heads, T for tails (e.g., two possible results are HTHT or THTH). How many results are possible?

(A) 4

(B) 8

(C) 16

(D) 32