Predicate Logic

CS 130

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Predicates and Terms

Definition (Predicate)

- A "yes/no", "true/false" property of one or more terms
- Typically represented by uppercase letters (A, B, C, etc.)

Definition (Term)

- A variable, denoted by a lowercase letter (a, b, c, etc.)
- A constant, denoted by an underlined lowercase letter ($\underline{a}$, $\underline{b}$, $\underline{c}$, etc.)
- A function, also denoted by a lowercase letter, but followed by a comma-separated list of more terms—inputs

Predicates and Terms

Examples (Notation)

Typically, we write predicates and functions in prefix notation:

- $A(x)$
- $B(x, f(y), \underline{z})$
- $q(n, \underline{m})$
- $g(x, \underline{a}, h(z))$

For common predicates like $=$, we might also write infix notation:

$$x = y \qquad \text{VS} \qquad = (x, y)$$

Example (Predicates vs Functions)

Compare:

- $f(x)$: "the father of x"
- $F(x)$: "x is a father"

Multiple Choice Question

Let

$$K(x) = \text{“}x \text{ is a Klingon”}$$
$$V(x) = \text{“}x \text{ is a Vulcan”}$$
$$f(x) = \text{the father of } x$$
$$\underline{s} = \text{Spock}$$
$$\underline{w} = \text{Worf}$$

How would you symbolize the sentence “Worf is a Klingon”?

(A) $K(\underline{w})$

(B) $K(x)$

(C) $K(w)$

(D) $\underline{w}(K)$

Multiple Choice Question

Let

$$
\begin{aligned}
K(x) &= \text{“}x \text{ is a Klingon”} \\
V(x) &= \text{“}x \text{ is a Vulcan”} \\
f(x) &= \text{the father of } x \\
\underline{s} &= \text{Spock} \\
\underline{w} &= \text{Worf}
\end{aligned}
$$

How would you symbolize the sentence “Worf is a Vulcan”?

(A) $K(\underline{w})$

(B) $V(\underline{s})$

(C) $V(\underline{w})$

(D) You can't; Worf is a Klingon, not a Vulcan

Multiple Choice Question

Let

$$K(x) = \text{“}x \text{ is a Klingon”}$$
$$V(x) = \text{“}x \text{ is a Vulcan”}$$
$$f(x) = \text{the father of } x$$
$$\underline{s} = \text{Spock}$$
$$\underline{w} = \text{Worf}$$

How would you symbolize the sentence “Spock’s father is a Vulcan”?

(A) $f(V(\underline{s}))$

(B) $V(\underline{s})$

(C) $V(f(\underline{s}))$

(D) $V(x)$

Multiple Choice Question

Let

$$K(x) = \text{“}x \text{ is a Klingon”}$$
$$V(x) = \text{“}x \text{ is a Vulcan”}$$
$$f(x) = \text{the father of } x$$
$$\underline{s} = \text{Spock}$$
$$\underline{w} = \text{Worf}$$

How would you symbolize the sentence “Darth Vader is a Vulcan”?

(A) $V(\underline{d})$

(B) $V(\text{Darth Vader})$

(C) $V(x)$

(D) None of the above

Quantifiers

Definition

We will have two quantifier symbols, $\forall$ and $\exists$, which will always be followed by a variable (never a constant or a function)

- $\forall x$ is pronounced "for all x"
- $\exists x$ is pronounced "there exists an x"

Predicate WFFs

Definition

One of:

- A term
- A predicate applied to one or more terms
- $(F_1 \wedge F_2)$
- $(F_1 \vee F_2)$
- $(F_1 \implies F_2)$
- $(F_1 \iff F_2)$
- $\neg F_1$
- $\forall x[F_1]$
- $\exists x[F_1]$

where F_1 and F_2 are themselves predicate WFFs and x is any variable.

Predicate WFFs

Note

- We'll often denote an arbitrary predicate WFF by a single uppercase letter (A, B, C, etc.)
- We'll usually use $A(x, y, z)$ to represent a predicate WFF involving the variables x, y, and z

If there's any potential confusion between a formula and a predicate, I'll try to have an extra explanation.

Multiple Choice Question

Let

$$M(x) = \text{“}x \text{ is a Class M planet”}$$
$$L(x) = \text{“}x \text{ supports life”}$$

What is the English equivalent of the following predicate WFF?

$$\forall x[(L(x) \implies M(x))]$$

(A) Every Class M planet supports life
(B) Every life-supporting planet is Class M
(C) Some Class M planets do not support life
(D) All planets support life or are Class M

Multiple Choice Question

Let

$$M(x) = \text{“}x \text{ is a Class M planet”}$$
$$L(x) = \text{“}x \text{ supports life”}$$

What is the English equivalent of the following predicate WFF?

$$\forall x[(M(x) \vee L(x))]$$

(A) All planets are either Class M or support life

(B) Either all planets are Class M or all planets support life

(C) All planets are Class M and support life

(D) Some planets are Class M; others support life

Multiple Choice Question

Let

$$M(x) = \text{“}x \text{ is a Class M planet”}$$
$$L(x) = \text{“}x \text{ supports life”}$$

What is the English equivalent of the following predicate WFF?

$$(\forall x[M(x)] \vee \forall x[L(x)])$$

(A) All planets are either Class M or support life

(B) Either all planets are Class M or all planets support life

(C) All planets are Class M and support life

(D) Some planets are Class M; others support life

Multiple Choice Question

Let

$$M(x) = \text{“}x \text{ is a Class M planet”}$$
$$L(x) = \text{“}x \text{ supports life”}$$

What is the English equivalent of the following predicate WFF?

$$\forall x[(M(x) \vee \neg M(x))]$$

(A) All planets are Class M

(B) All planets are Class M, or all planets are not

(C) Some planets are Class M; others are not

(D) A planet is either Class M or it is not

Multiple Choice Question

Let

$$M(x) = \text{“}x \text{ is a Class M planet”}$$
$$L(x) = \text{“}x \text{ supports life”}$$

What is the English equivalent of the following predicate WFF?

$$(\forall x[M(x)] \lor \forall x[\neg M(x)])$$

(A) Some planets are Class M; others are not

(B) All planets are Class M, or all planets are not

(C) Some planets are Class M and others are not

(D) A planet is either Class M or it is not

Multiple Choice Question

Let

$$M(x) = \text{“}x \text{ is a Class M planet”}$$
$$L(x) = \text{“}x \text{ supports life”}$$

What is the English equivalent of the following predicate WFF?

$$\neg\forall x[(M(x) \vee L(x))]$$

(A) Some planets are neither Class M nor support life

(B) Not all planets are both Class M and support life

(C) All planets are neither Class M nor support life

(D) All planets are either not Class M or do not support life

Thinking About $\forall$ and $\exists$

$\forall$ is like a big conjunction

Intuitively, $\forall x[x > 2]$ means:

$$\begin{aligned} & 0 > 2 \\ \wedge\ & 1 > 2 \\ \wedge\ & 2 > 2 \\ \wedge\ & 3 > 2 \\ & \vdots \end{aligned}$$

$\exists$ is like a big disjunction

Intuitively, $\exists x[x > 2]$ means:

$$\begin{aligned} & 0 > 2 \\ \vee\ & 1 > 2 \\ \vee\ & 2 > 2 \\ \vee\ & 3 > 2 \\ & \vdots \end{aligned}$$

A	B	$(\neg(A$	$\wedge$	$B)$	$\iff$	$(\neg A$	$\vee$	$\neg B))$
F	F	T	F		T	T	T	T
F	T	T	F		T	T	T	F
T	F	T	F		T	F	T	T
T	T	F	T		T	F	F	F

Thinking About $\forall$ and $\exists$

$\forall$ is like a big conjunction

Intuitively, $\forall x[x > 2]$ means:

$$
\begin{aligned}
& 0 > 2 \\
\wedge\ & 1 > 2 \\
\wedge\ & 2 > 2 \\
\wedge\ & 3 > 2 \\
& \vdots
\end{aligned}
$$

$\exists$ is like a big disjunction

Intuitively, $\exists x[x > 2]$ means:

$$
\begin{aligned}
& 0 > 2 \\
\vee\ & 1 > 2 \\
\vee\ & 2 > 2 \\
\vee\ & 3 > 2 \\
& \vdots
\end{aligned}
$$

A	B	$(\neg(A$	$\vee$	$B)$	$\iff$	$(\neg A$	$\wedge$	$\neg B))$
F	F	T	F		T	T	T	T
F	T	F	T		T	T	F	F
T	F	F	T		T	F	F	T
T	T	F	T		T	F	F	F

Multiple Choice Question

Let $L(x, y) =$ "x loves y".
What is the predicate logic equivalent of the following English sentence?

Somebody loves everybody

(A) $\forall x[\forall y[L(x, y)]]$
(B) $\forall x[\exists y[L(x, y)]]$
(C) $\exists x[\forall y[L(x, y)]]$
(D) $\exists x[\exists y[L(x, y)]]$

Multiple Choice Question

Let $L(x, y) =$ "x loves y".
What is the predicate logic equivalent of the following English sentence?

Everybody loves somebody

(A) $\forall x[\forall y[L(x, y)]]$
(B) $\forall x[\exists y[L(x, y)]]$
(C) $\exists x[\forall y[L(x, y)]]$
(D) $\exists x[\exists y[L(x, y)]]$

Multiple Choice Question

Let $L(x, y) =$ "x loves y".
What is the predicate logic equivalent of the following English sentence?

Somebody loves somebody

(A) $\forall x[\forall y[L(x, y)]]$
(B) $\forall x[\exists y[L(x, y)]]$
(C) $\exists x[\forall y[L(x, y)]]$
(D) $\exists x[\exists y[L(x, y)]]$

Multiple Choice Question

Let $L(x, y) =$ "x loves y".
What is the predicate logic equivalent of the following English sentence?

Everybody loves everybody

(A) $\forall x[\forall y[L(x, y)]]$
(B) $\forall x[\exists y[L(x, y)]]$
(C) $\exists x[\forall y[L(x, y)]]$
(D) $\exists x[\exists y[L(x, y)]]$

Multiple Choice Question

Let $L(x, y) =$ "x loves y".
What is the predicate logic equivalent of the following English sentence?

Everybody is loved by everybody

(A) $\forall x[\forall y[L(y, x)]]$
(B) $\forall x[\forall y[L(x, y)]]$
(C) $\exists x[\exists y[L(x, y)]]$
(D) $\exists x[\exists y[L(y, x)]]$

Multiple Choice Question

Let $L(x, y) =$ "x loves y".
What is the predicate logic equivalent of the following English sentence?

Somebody is loved by somebody

(A) $\forall x[\forall y[L(y, x)]]$
(B) $\forall x[\forall y[L(x, y)]]$
(C) $\exists x[\exists y[L(x, y)]]$
(D) $\exists x[\exists y[L(y, x)]]$

Multiple Choice Question

Let $L(x, y) =$ "x loves y".
What is the predicate logic equivalent of the following English sentence?

Everybody is loved by somebody

(A) $\forall x[\exists y[L(y, x)]]$
(B) $\forall x[\exists y[L(x, y)]]$
(C) $\exists x[\forall y[L(x, y)]]$
(D) $\exists x[\forall y[L(y, x)]]$

Multiple Choice Question

Let $L(x, y) =$ "x loves y".
What is the predicate logic equivalent of the following English sentence?

Somebody is loved by everybody

(A) $\forall x[\exists y[L(y, x)]]$
(B) $\forall x[\exists y[L(x, y)]]$
(C) $\exists x[\forall y[L(x, y)]]$
(D) $\exists x[\forall y[L(y, x)]]$

Multiple Choice Question

Let

$$P(x) = \text{“}x \text{ is a man”}$$
$$Q(x) = \text{“}x \text{ is mortal”}$$
$$\underline{s} = \text{Socrates}$$

How would you symbolize the following argument in predicate logic?

All men are mortal.
Socrates is a man.

∴ Socrates is mortal.

(A) $\forall x[(P(x) \wedge Q(x))], \quad P(\underline{s}) \vdash Q(\underline{s})$
(B) $\forall x[(P(x) \wedge Q(x))], \quad \exists \underline{s}[P(\underline{s})] \vdash Q(\underline{s})$
(C) $\forall x[(P(x) \Longrightarrow Q(x))], \quad P(\underline{s}) \vdash Q(\underline{s})$
(D) $\forall x[(P(x) \Longrightarrow Q(x))], \quad \exists \underline{s}[P(\underline{s})] \vdash Q(\underline{s})$

Multiple Choice Question

Is the following true or false?

$$x = 2$$

(A) True

(B) False

(C) Can't be determined

Free Variables

Definition (Scope)

The scope of a quantifier is the stuff in between the square brackets:

$$\forall x \overbrace{[\qquad \ldots \qquad]}^{\text{scope of } \forall x}$$

$$\exists x \underbrace{[\qquad \ldots \qquad]}_{\text{scope of } \exists x}$$

Definition (Free & Bound Variables)

A variable in a predicate WFF is bound if it occurs within the scope of a quantifier over that variable.
Otherwise, the variable is free.

Free Variables

Examples

Free variables are highlighted in the WFFs below:

- $\Big(\forall x\big[(P(x) \implies \forall y[(R(x) \vee Q(y))]\big)\big] \wedge B(x)\Big)$
- $\Big(\exists x\big[\forall y[M(x,\ y,\ f(x, z))]\big] \vee G(x,\ y,\ z)\Big)$

Definition

A closed predicate WFF is one in which there are no free variables.
Also known as a sentence.

Multiple Choice Question

Which of the following is a closed formula?

(A) $\forall x[x = 2]$

(B) $\exists x[x = 2]$

(C) Both of the above

(D) None of the above

Multiple Choice Question

Which of the following is true?

(A) $\forall x[x = 2]$

(B) $\exists x[x = 2]$

(C) Both of the above

(D) None of the above

Models

Definition

To evaluate the truth of a predicate WFF, we may need to define a model (aka an interpretation):

- A "universe" (or domain of interpretation) of possible objects that variables may represent
- The meanings of all predicate symbols
- The meanings of all function symbols
- The meanings of all constant symbols (which must represent objects from the domain of interpretation)

Note

- We don't change the meaning of symbols like $\forall$, $\exists$, $\wedge$, $\vee$, etc.
- All predicates and functions must make sense for everything in the particular universe

Multiple Choice Question

$$\left(\forall x[(P(x) \vee Q(x))] \implies (\forall x[P(x)] \vee \forall x[Q(x)])\right)$$

Given the following model, is the WFF true or false?

- The universe is everybody in this room
- $P(x)$ = "x is a student"
- $Q(x)$ = "x is a teacher"

(A) True

(B) False

(C) Can't be determined

Multiple Choice Question

$$\left(\forall x[(P(x) \vee Q(x))] \implies (\forall x[P(x)] \vee \forall x[Q(x)])\right)$$

Given the following model, is the WFF true or false?

- The universe is everybody in this room
- $P(x)$ = "x is wearing clothes"
- $Q(x)$ = "x is wearing shoes"

(A) True

(B) False

(C) Can't be determined

Multiple Choice Question

$$\left(\forall x[(P(x) \lor Q(x))] \implies (\forall x[P(x)] \lor \forall x[Q(x)])\right)$$

Given the following model, is the WFF true or false?

- The universe is everybody on Earth
- $P(x) =$ "x is wearing clothes"
- $Q(x) =$ "x is wearing shoes"

(A) True

(B) False

(C) Can't be determined

Multiple Choice Question

Can we determine the truth value of a predicate WFF without a model?

(A) Yes

(B) No

Validity

Definition

A predicate WFF is valid if it is true in every possible model

Examples

- $\forall x[(A(x) \implies A(x))]$
- $\forall x\Big[\exists y\big[(C(x,y) \implies C(x,y))\big]\Big]$
- $\Big(\forall x[(P(x) \wedge Q(x))] \iff (\forall x[P(x)] \wedge \forall x[Q(x)])\Big)$
- $\Big(\forall x[W] \iff \neg\exists x[\neg W]\Big)$
- $\Big(\exists y[A] \implies (\forall x[B] \implies \exists y[A])\Big)$

Moral: all propositional tautologies are logically valid predicate WFFs, but not every logically valid predicate WFF is an instance of a propositional tautology.

Predicate Logic Proof System

Definition (Axioms)

Axiom 1: $\vdash \quad (A \implies (B \implies A))$

Axiom 2: $\vdash \quad ((A \implies (B \implies C)) \implies ((A \implies B) \implies (A \implies C)))$

Axiom 3: $\vdash \quad ((\neg B \implies \neg A) \implies (A \implies B))$

Axiom 4: $\vdash \quad (\forall x[A(x)] \implies A(t))$,
Provided that: x can be replaced by term t in $A(x)$

Axiom 5: $\vdash \quad (\forall x[(A \implies B)] \implies (A \implies \forall x[B]))$,
Provided that: x does not occur free in A

Definition (Inference Rules)

$$A, \quad (A \implies B) \quad \vdash \quad B \qquad \textbf{(Modus Ponens)}$$

$$A \quad \vdash \quad \forall x[A] \qquad \textbf{(Universal Generalization)}$$

When Can We Replace A Variable With A Term?

Axiom 4: $\vdash \quad (\forall x[A(x)] \implies A(t))$,
Provided that: x can be replaced by term t in $A(x)$

- For every variable v in term $t\ldots$
- $\ldots x$ must not occur free within the scope of a $\forall v$ or $\exists v$ in $A(x)$

Examples

Is the following a correct instance of Axiom 4?

$$\left(\forall x\left[\exists y[B(x,y,z)]\right] \implies \exists y[B(x,y,z)]\right)$$

(A) Yes

(B) No

When Can We Replace A Variable With A Term?

Axiom 4: $\vdash \quad (\forall x[A(x)] \implies A(t))$,
Provided that: x can be replaced by term t in $A(x)$

- For every variable v in term t...
- ... x must not occur free within the scope of a $\forall v$ or $\exists v$ in $A(x)$

Examples

Is the following a correct instance of Axiom 4?

$$\Big(\forall x\big[\exists y[B(x,y,z)]\big] \implies \exists y[B(x,y,z)]\Big)$$

(A) Yes —there are no $\exists x$ or $\forall x$ quantifiers in $\exists y[B(x,y,z)]$

(B) No

When Can We Replace A Variable With A Term?

Axiom 4: $\vdash \quad (\forall x[A(x)] \implies A(t)),$

Provided that: x can be replaced by term t in $A(x)$

- For every variable v in term t...
- ...x must not occur free within the scope of a $\forall v$ or $\exists v$ in $A(x)$

Examples

Is the following a correct instance of Axiom 4?

$$\Big(\forall x\big[\exists y[B(x,y,z)]\big] \implies \exists y[B(y,y,z)]\Big)$$

(A) Yes

(B) No

When Can We Replace A Variable With A Term?

Axiom 4: $\vdash \quad (\forall x[A(x)] \implies A(t))$,
Provided that: x can be replaced by term t in $A(x)$

- For every variable v in term t...
- ...x must not occur free within the scope of a $\forall v$ or $\exists v$ in $A(x)$

Examples

Is the following a correct instance of Axiom 4?

$$\Big(\forall x\big[\exists y[B(x,y,z)]\big] \implies \exists y[B(y,y,z)]\Big)$$

(A) Yes

(B) No—x occurs free in the scope of $\exists y$

When Can We Replace A Variable With A Term?

Axiom 4: $\vdash \quad (\forall x[A(x)] \implies A(t))$,
Provided that: x can be replaced by term t in $A(x)$

- For every variable v in term t...
- ... x must not occur free within the scope of a $\forall v$ or $\exists v$ in $A(x)$

Examples

Is the following a correct instance of Axiom 4?

$$\left(\forall x\left[\exists y[B(x,y,z)]\right] \implies \exists y[B(z,y,z)]\right)$$

(A) Yes

(B) No

When Can We Replace A Variable With A Term?

Axiom 4: $\vdash \quad (\forall x[A(x)] \implies A(t)),$

Provided that: x can be replaced by term t in $A(x)$

- For every variable v in term $t\ldots$
- $\ldots x$ must not occur free within the scope of a $\forall v$ or $\exists v$ in $A(x)$

Examples

Is the following a correct instance of Axiom 4?

$$\Big(\forall x\big[\exists y[B(x,y,z)]\big] \implies \exists y[B(z,y,z)]\Big)$$

(A) Yes —there are no $\exists z$ or $\forall z$ quantifiers in $\exists y[B(x,y,z)]$

(B) No

When Can We Replace A Variable With A Term?

Axiom 4: $\vdash \quad (\forall x[A(x)] \implies A(t)),$
Provided that: x can be replaced by term t in $A(x)$

- For every variable v in term $t\ldots$
- $\ldots x$ must not occur free within the scope of a $\forall v$ or $\exists v$ in $A(x)$

Examples

Is the following a correct instance of Axiom 4?

$$\left(\forall x\left[\exists y[B(x,y,z)]\right] \implies \exists y[B(q,y,z)]\right)$$

(A) Yes

(B) No

When Can We Replace A Variable With A Term?

Axiom 4: $\vdash \quad (\forall x[A(x)] \implies A(t))$,
Provided that: x can be replaced by term t in $A(x)$

- For every variable v in term t...
- ... x must not occur free within the scope of a $\forall v$ or $\exists v$ in $A(x)$

Examples

Is the following a correct instance of Axiom 4?

$$\Big(\forall x\big[\exists y[B(x,y,z)]\big] \implies \exists y[B(q,y,z)]\Big)$$

(A) Yes —there are no $\exists q$ or $\forall q$ quantifiers in $\exists y[B(x,y,z)]$

(B) No

When Can We Replace A Variable With A Term?

Axiom 4: $\vdash \quad (\forall x[A(x)] \implies A(t))$,
Provided that: x can be replaced by term t in $A(x)$

- For every variable v in term t...
- ... x must not occur free within the scope of a $\forall v$ or $\exists v$ in $A(x)$

Examples

Is the following a correct instance of Axiom 4?

$$\left(\forall x\left[\exists y[B(x,y,z)]\right] \implies \exists y[B(\underline{c},y,z)]\right)$$

(A) Yes

(B) No

When Can We Replace A Variable With A Term?

Axiom 4: $\vdash \quad (\forall x[A(x)] \implies A(t))$,
Provided that: x can be replaced by term t in $A(x)$

- For every variable v in term $t\ldots$
- $\ldots x$ must not occur free within the scope of a $\forall v$ or $\exists v$ in $A(x)$

Examples

Is the following a correct instance of Axiom 4?

$$\left(\forall x\left[\exists y[B(x,y,z)]\right] \implies \exists y[B(\underline{c},y,z)]\right)$$

(A) Yes —there are no variables in the term $\underline{c}$

(B) No

When Can We Replace A Variable With A Term?

Axiom 4: $\vdash \quad (\forall x[A(x)] \implies A(t))$,
Provided that: x can be replaced by term t in $A(x)$

- For every variable v in term t...
- ... x must not occur free within the scope of a $\forall v$ or $\exists v$ in $A(x)$

Examples

Is the following a correct instance of Axiom 4?

$$\Big(\forall x\big[\exists y[B(x,y,z)]\big] \implies \exists y[B(f(x,y),y,z)]\Big)$$

(A) Yes

(B) No

When Can We Replace A Variable With A Term?

Axiom 4: $\vdash \quad (\forall x[A(x)] \implies A(t))$,
Provided that: x can be replaced by term t in $A(x)$

- For every variable v in term t...
- ...x must not occur free within the scope of a $\forall v$ or $\exists v$ in $A(x)$

Examples

Is the following a correct instance of Axiom 4?

$$\left(\forall x\left[\exists y[B(x,y,z)]\right] \implies \exists y[B(f(x,y),y,z)]\right)$$

(A) Yes

(B) No —x occurs free in the scope of $\exists y$, and y is a variable in the term $f(x,y)$

Multiple Choice Question

Axiom 5: $\vdash \quad (\forall x[(A \implies B)] \implies (A \implies \forall x[B])),$
Provided that: x does not occur free in A

Is the following a correct instance of Axiom 5?

$$\left(\forall x\left[(\exists y[A(x,y)] \implies B(x))\right] \implies (\exists y[A(x,y)] \implies \forall x[B(x)])\right)$$

(A) Yes

(B) No

Abbreviations and Notation

Definitions

Like in propositional logic, we use the following abbreviations

$$
\begin{aligned}
(A \wedge B) &\quad \text{abbreviates} \quad \neg(A \implies \neg B) \\
(A \vee B) &\quad \text{abbreviates} \quad (\neg A \implies B) \\
(A \iff B) &\quad \text{abbreviates} \quad \neg((A \implies B) \implies \neg(B \implies A))
\end{aligned}
$$

Additionally,

$$\exists x[W] \quad \text{abbreviates} \quad \neg \forall x[\neg W]$$

Metatheorems

Theorem (Soundness)

If: *A is logically valid*

Then: $\vdash \quad A$

Theorem (Completeness)

If: $\vdash \quad A$

Then: *A is logically valid*

Theorem (The Deduction Theorem)

If: $G_1, \quad G_2, \quad \ldots, \quad G_n, \quad A \quad \vdash \quad B$

Provided that: in the proof, there are no applications of Universal Generalization to any free variables in A

Then: $G_1, \quad G_2, \quad \ldots, \quad G_n \quad \vdash \quad (A \implies B)$

Multiple Choice Question

Does the converse of the Deduction Theorem still hold?

(A) Yes

(B) No