Propositional Logic

CS 130

Alex Vondrak

ajvondrak@csupomona.edu

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Propositional Logic

Definition

A proposition is a statement that is either true or false.

Examples

The following are propositions:

- 2 is prime . . . is true
- Earth is the center of the universe . . . is false
- Powdered non-dairy creamer is flammable . . . is true
- Macs can't get malware . . . is false

Propositional Logic

Definition

A proposition is a statement that is either true or false.

Examples

The following are not propositions:

- 2 . . . ?
- Is Earth is the center of the universe? . . . ?
- Light non-dairy creamer on fire . . . ?
- Macs can't . . . ?

Multiple Choice Question

Which of the following is a proposition?

(A) $\cos(x) = -1$

(B) Life exists on other planets

(C) Will the game be over soon?

(D) Add 5 to both sides

Multiple Choice Question

Which of the following is not a proposition?

(A) The moon is made of green cheese

(B) In the beginning God created the heaven and the earth

(C) Buy low, sell high

(D) William Shatner has tinnitus

Connectives

Much as we can form compound sentences in English, we can form compound propositions using connectives (aka operators).

Examples

- "It is not the case that ______________."
- "______________ and ______________."
- "______________ or ______________."
- "If ______________, then ______________."
- "______________ if and only if ______________."

Multiple Choice Question

Which of the following is a compound proposition?

(A) A traditional communist symbol is the hammer and sickle

(B) World War II occurred between the years 1939 and 1945

(C) “With liberty and justice for all” is the last line of the Pledge of Allegiance

(D) Australia, Great Britain, and Switzerland have sent a team to every Olympic Games

Variables

Instead of writing out entire propositions, we'll use variables

- Shorter to write
- Abstracts away details
- Allows us to analyze propositions at a high level

Convention

- Use lowercase letters for simple propositions (a, b, c, etc.)
- Use UPPERCASE letters for compound propositions (A, B, C, etc.)

Truth Tables

Definition

A truth table displays the truth value of a compound proposition for every possible value of its constituent propositions

Example

We can make a table that describes *any* compound proposition of the form

"It is not the case that ________________"

by making a table: for every truth value the blank might have, we can derive the truth value of the entire compound proposition (i.e., it is truth-functional)

Negation ("It is not the case that ________")

Definition (Notation)

Given any proposition (compound or otherwise), A, its negation is denoted by any of the following:

- $\neg A$
- A'
- $!A$
- $\sim A$
- $\overline{A}$

In this class, we'll use the first notation and pronounce it "not A"

Multiple Choice Question

A	$\neg A$

How many possible truth values does A have?

(A) 2

(B) 4

(C) ∞

(D) None of the above

Multiple Choice Question

A	$\neg A$
F	
T	

If A is F, what is the truth value of $\neg A$?

(A) T

(B) F

Multiple Choice Question

A	$\neg A$
F	T
T	

If A is T, what is the truth value of $\neg A$?

(A) T

(B) F

Negation ("It is not the case that ________")

Definition

A	$\neg A$
F	T
T	F

Conjunction ("________ and ________")

Definition (Notation)

Given any two propositions (compound or otherwise), A and B, their conjunction is denoted by any of the following:

- $A \wedge B$
- A & B
- A && B
- $A \cdot B$

In this class, we'll use the first notation and pronounce it "A and B"; in a conjunction, A and B are called conjuncts

Multiple Choice Question

A	B	$A \wedge B$

How many different ways are there to pick truth values for A and B?

(A) 2

(B) 4

(C) 8

(D) None of the above

Multiple Choice Question

$$\overbrace{A \quad B \quad \cdots}^{v \text{ variables}} \;\Big|\; A \text{ op } B \text{ op } \cdots$$

A	B	$\cdots$	A op B op $\cdots$

How many rows r are in a truth table with v variables?

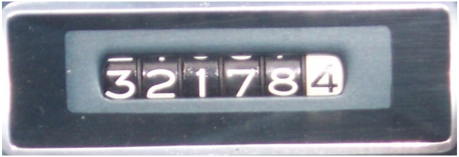
(A) $r = 2^v$

(B) $r = v^2$

(C) $r = 2 \times v$

(D) $r = 2 + v$

Multiple Choice Question



The odometer pictured has six digits—“columns” of spinners labeled 0–9. At what value does it start?

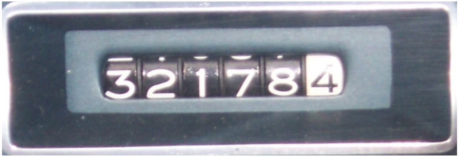
(A) 0

(B) 321784

(C) 000000

(D) 00000.0

Multiple Choice Question



The odometer pictured has six digits—"columns" of spinners labeled 0–9. At what value can we no longer increment the odometer?

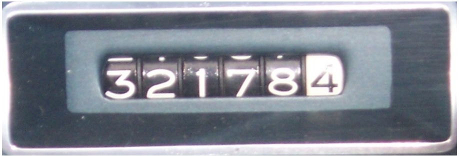
(A) 999999

(B) 99999.9

(C) 100000

(D) No such limit

Multiple Choice Question



The odometer pictured has six digits—"columns" of spinners labeled 0–9. When the odometer increments from 000000, which column spins first?

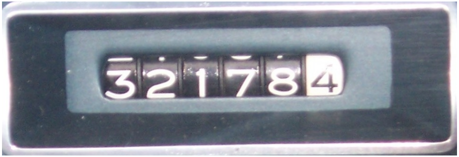
(A) The leftmost

(B) The rightmost

(C) Second from the left

(D) Second from the right

Multiple Choice Question



The odometer pictured has six digits—"columns" of spinners labeled 0–9. Suppose we keep incrementing the odometer. When does the second ticker from the right spin?

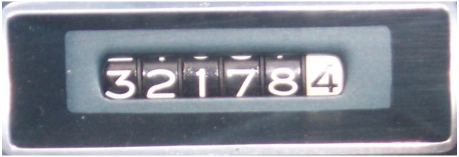
(A) Depends on how high we've incremented

(B) On each increment

(C) As long as the rightmost column is 0

(D) Whenever the rightmost column transitions from 9 to 0

Multiple Choice Question



The odometer pictured has six digits—"columns" of spinners labeled 0–9. Now imagine there are only two columns of spinner labeled 0–1.
At what value do we start?

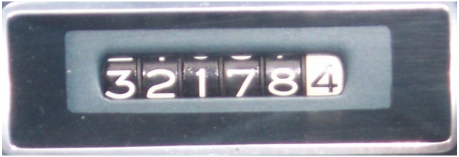
(A) 00

(B) 01

(C) 10

(D) 11

Multiple Choice Question



The odometer pictured has six digits—“columns” of spinners labeled 0–9. Now imagine there are only two columns of spinner labeled 0–1. At what value do we stop?

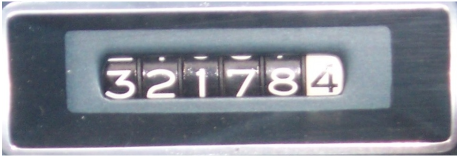
(A) 00

(B) 01

(C) 10

(D) 11

Multiple Choice Question



The odometer pictured has six digits—“columns” of spinners labeled 0–9. Now imagine there are only two columns of spinner labeled 0–1. Starting at 00, increment the odometer once. What does it read now?

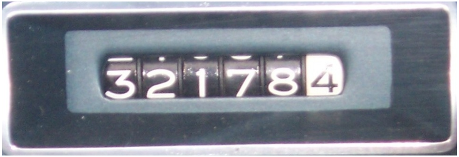
(A) 00

(B) 01

(C) 10

(D) 11

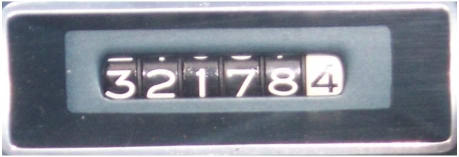
Multiple Choice Question



The odometer pictured has six digits—"columns" of spinners labeled 0–9. Now imagine there are only two columns of spinner labeled 0–1. At 01, increment the odometer once. What does it read now?

(A) 00
(B) 01
(C) 10
(D) 11

Multiple Choice Question



The odometer pictured has six digits—"columns" of spinners labeled 0–9.
Now imagine there are only two columns of spinner labeled 0–1.
At 10, increment the odometer once. What does it read now?

(A) 00

(B) 01

(C) 10

(D) 11

Conjunction ("__________ and __________")

Definition (In Progress)

A	B	$A \wedge B$
F	F	
F	T	
T	F	
T	T	

Note

If you do not use this "odometer order" for your truth tables on the homework/exams, you will receive no credit for that problem.
It's Draconian, but it helps both of us.

Multiple Choice Question

A	B	$A \wedge B$
F	F	
F	T	
T	F	
T	T	

When A is F and B is F, what should be the value of $A \wedge B$?

(A) F

(B) T

Multiple Choice Question

A	B	$A \wedge B$
F	F	F
F	T	
T	F	
T	T	

When A is F and B is T, what should be the value of $A \wedge B$?

(A) F

(B) T

Multiple Choice Question

A	B	$A \wedge B$
F	F	F
F	T	F
T	F	
T	T	

When A is T and B is F, what should be the value of $A \wedge B$?

(A) F

(B) T

Multiple Choice Question

A	B	$A \wedge B$
F	F	F
F	T	F
T	F	F
T	T	

When A is T and B is T, what should be the value of $A \wedge B$?

(A) F

(B) T

Conjunction ("________ and ________")

Definition

A	B	$A \wedge B$
F	F	F
F	T	F
T	F	F
T	T	T

Long story short: a conjunction is true only in the case when the conjuncts are both true.

Disjunction ("__________ or __________")

Definition

Denote the disjunction of two propositions A and B as $A \vee B$ and pronounce it "A or B"; A and B are called disjuncts

Other notations include $A + B$, $A \mid B$, and $A \parallel B$

A	B	$A \vee B$
F	F	
F	T	
T	F	
T	T	

(A) F

(B) T

Disjunction ("__________ or __________")

Definition

Denote the disjunction of two propositions A and B as $A \vee B$ and pronounce it "A or B"; A and B are called disjuncts

Other notations include $A + B$, $A \mid B$, and $A \parallel B$

A	B	$A \vee B$
F	F	F
F	T	
T	F	
T	T	

(A) F

(B) T

Disjunction ("__________ or __________")

Definition

Denote the disjunction of two propositions A and B as $A \vee B$ and pronounce it "A or B"; A and B are called disjuncts
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A	B	$A \vee B$
F	F	F
F	T	T
T	F	
T	T	

(A) F

(B) T

Disjunction ("__________ or __________")

Definition

Denote the disjunction of two propositions A and B as $A \vee B$ and pronounce it "A or B"; A and B are called disjuncts
Other notations include $A + B$, $A \mid B$, and $A \parallel B$

A	B	$A \vee B$
F	F	F
F	T	T
T	F	T
T	T	

(A) F

(B) T

Disjunction ("________ or ________")

Definition

Denote the disjunction of two propositions A and B as $A \vee B$ and pronounce it "A or B"; A and B are called disjuncts
Other notations include $A + B$, $A \mid B$, and $A \parallel B$

A	B	$A \vee B$
F	F	F
F	T	T
T	F	T
T	T	T

(A) F

(B) T

Inclusive vs Exclusive Or

Note

- The English "or" is often used as an exclusive or, meaning that one or the other is true, but **not** both
- By default, mathematicians use an inclusive or, meaning that **at least** one operand is true
- Exclusive-or ("xor") is typically denoted $A \oplus B$

Implication ("If ________, then ________")

Definition (Notation)

- $A \implies B$
- $A \longrightarrow B$
- $A \supset B$

Pronounced "A implies B" or "if A, then B"

A is called the...	B is called the...
antecedent	consequent
hypothesis	conclusion
premise	outcome
sufficient condition	necessary condition

Multiple Choice Question

Consider the following implication:

If you are male, then you are a bachelor

Is this implication false or true?

(A) `F`

(B) `T`

Multiple Choice Question

Consider the following implication:

If you are a bachelor, then you are male

Is this implication false or true?

(A) `F`

(B) `T`

Implication ("If ______, then ______")

Definition

A	B	$A \implies B$
F	F	
F	T	
T	F	
T	T	

(A) F

(B) T

Implication (“If ____________, then ____________”)

Definition

A	B	$A \implies B$
F	F	T
F	T	
T	F	
T	T	

(A) F

(B) T

Implication ("If ________, then ________")

Definition

A	B	$A \implies B$
F	F	T
F	T	T
T	F	
T	T	

(A) F

(B) T

Implication ("If ____________, then ____________")

Definition

A	B	$A \implies B$
F	F	T
F	T	T
T	F	F
T	T	

(A) F

(B) T

Implication (“If ________, then ________”)

Definition

A	B	$A \implies B$
F	F	T
F	T	T
T	F	F
T	T	T

Note

This is a common truth table to forget. An easy way to remember it: $A \implies B$ is the same as

$$A \leq B$$

where $\mathtt{F} = 0, \mathtt{T} = 1$. “Falser than or equal to”, if you will.

Biconditional ("________ if and only if ________")

Definition (Notation)

- $A \iff B$
- $A \longleftrightarrow B$
- $A \equiv B$

Pronounced "A is equivalent to B" or "A if and only if B"
When written, the shorthand "iff" is often used instead of "if and only if"

Definition

A	B	$A \iff B$
F	F	T
F	T	F
T	F	F
T	T	T

Well-Formed Formulae

Definition

A propositional well-formed formula (WFF) has one of the following forms:

- A variable for a simple proposition (a, b, c, etc.)
- A variable for a compound proposition (A, B, C, etc.)
- $\neg W_1$
- $(W_1 \wedge W_2)$
- $(W_1 \vee W_2)$
- $(W_1 \implies W_2)$
- $(W_1 \iff W_2)$

where W_1 and W_2 are themselves WFFs

Multiple Choice Question

Which of the following is a WFF?

(A) $((\neg A) \implies (\neg B))$

(B) $(\neg(A) \implies \neg(B))$

(C) $\neg(A \implies \neg B)$

(D) $\neg A \implies (\neg B)$

Multiple Choice Question

Which of the following is not a WFF?

(A) $\neg\neg A$

(B) p

(C) $((a \implies B) \implies (C \implies d))$

(D) $((a \iff a) \vee a) \wedge a)$

Multiple Choice Question

Let

$$a = \text{“}\textit{Star Trek} \text{ is a documentary”}$$
$$b = \text{“}\textit{Star Trek} \text{ is a movie”}$$
$$c = \text{“There is life on another planet”}$$

Give the truth value of the WFF

$$(a \wedge b)$$

(A) F
(B) T
(C) Can't be sure

Multiple Choice Question

Let

$$a = \text{“\textit{Star Trek} is a documentary”}$$
$$b = \text{“\textit{Star Trek} is a movie”}$$
$$c = \text{“There is life on another planet”}$$

Give the truth value of the WFF

$$(a \vee b)$$

(A) F
(B) T
(C) Can't be sure

Multiple Choice Question

Let

$$a = \text{“\textit{Star Trek} is a documentary”}$$
$$b = \text{“\textit{Star Trek} is a movie”}$$
$$c = \text{“There is life on another planet”}$$

Give the truth value of the WFF

$$(b \vee c)$$

(A) F

(B) T

(C) Can't be sure

Multiple Choice Question

Let

$$a = \text{“\textit{Star Trek} is a documentary”}$$
$$b = \text{“\textit{Star Trek} is a movie”}$$
$$c = \text{“There is life on another planet”}$$

Give the truth value of the WFF

$$(a \iff c)$$

(A) F
(B) T
(C) Can't be sure

Complex Truth Tables

We may evaluate when a WFF is true by analyzing the truth values of every subexpression

Example (Biconditional Exchange)

A	B	$((A \iff B)$	$\iff$	$((A \implies B)$	$\wedge$	$(B \implies A)))$
F	F	?	?	?	?	?
F	T	?	?	?	?	?
T	F	?	?	?	?	?
T	T	?	?	?	?	?

(A) F

(B) T

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A	B	$((A \iff B)$	$\iff$	$((A \implies B)$	$\wedge$	$(B \implies A)))$
F	F	T	?	?	?	?
F	T	?	?	?	?	?
T	F	?	?	?	?	?
T	T	?	?	?	?	?

(A) F

(B) T

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We may evaluate when a WFF is true by analyzing the truth values of every subexpression

Example (Biconditional Exchange)

A	B	$((A$	$\iff$	$B)$	$\iff$	$((A$	$\implies$	$B)$	$\wedge$	$(B$	$\implies$	$A)))$
F	F		T		?		T		?		?	
F	T		?		?		?		?		?	
T	F		?		?		?		?		?	
T	T		?		?		?		?		?	

(A) F

(B) T

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We may evaluate when a WFF is true by analyzing the truth values of every subexpression

Example (Biconditional Exchange)

A	B	$((A$	$\iff$	$B)$	$\iff$	$((A$	$\implies$	$B)$	$\land$	$(B$	$\implies$	$A)))$
F	F		T		?		T		?		T	
F	T		?		?		?		?		?	
T	F		?		?		?		?		?	
T	T		?		?		?		?		?	

(A) F

(B) T

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We may evaluate when a WFF is true by analyzing the truth values of every subexpression

Example (Biconditional Exchange)

A	B	$((A \iff B)$	$\iff$	$((A \implies B)$	$\land$	$(B \implies A)))$
F	F	T	?	T	T	T
F	T	?	?	?	?	?
T	F	?	?	?	?	?
T	T	?	?	?	?	?

(A) F

(B) T

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We may evaluate when a WFF is true by analyzing the truth values of every subexpression

Example (Biconditional Exchange)

A	B	$((A \iff B)$	$\iff$	$((A \implies B)$	$\wedge$	$(B \implies A)))$
F	F	T	T	T	T	T
F	T	?	?	?	?	?
T	F	?	?	?	?	?
T	T	?	?	?	?	?

(A) F

(B) T

Complex Truth Tables

We may evaluate when a WFF is true by analyzing the truth values of every subexpression

Example (Biconditional Exchange)

A	B	$((A \iff B)$	$\iff$	$((A \implies B)$	$\wedge$	$(B \implies A)))$
F	F	T	T	T	T	T
F	T	F	?	?	?	?
T	F	?	?	?	?	?
T	T	?	?	?	?	?

(A) F

(B) T

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We may evaluate when a WFF is true by analyzing the truth values of every subexpression

Example (Biconditional Exchange)

A	B	$((A \iff B)$	$\iff$	$((A \implies B)$	$\wedge$	$(B \implies A)))$
F	F	T	T	T	T	T
F	T	F	?	T	?	?
T	F	?	?	?	?	?
T	T	?	?	?	?	?

(A) F

(B) T

Complex Truth Tables

We may evaluate when a WFF is true by analyzing the truth values of every subexpression

Example (Biconditional Exchange)

A	B	$((A \iff B)$	$\iff$	$((A \implies B)$	$\wedge$	$(B \implies A)))$
F	F	T	T	T	T	T
F	T	F	?	T	?	F
T	F	?	?	?	?	?
T	T	?	?	?	?	?

(A) F

(B) T

Complex Truth Tables

We may evaluate when a WFF is true by analyzing the truth values of every subexpression

Example (Biconditional Exchange)

A	B	$((A \iff B)$	$\iff$	$((A \implies B)$	$\land$	$(B \implies A)))$
F	F	T	T	T	T	T
F	T	F	?	T	F	F
T	F	?	?	?	?	?
T	T	?	?	?	?	?

(A) F

(B) T

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Example (Biconditional Exchange)

A	B	$((A \iff B)$	$\iff$	$((A \implies B)$	$\wedge$	$(B \implies A)))$
F	F	T	T	T	T	T
F	T	F	T	T	F	F
T	F	?	?	?	?	?
T	T	?	?	?	?	?

(A) F

(B) T

Complex Truth Tables

We may evaluate when a WFF is true by analyzing the truth values of every subexpression

Example (Biconditional Exchange)

A	B	$((A \iff B)$	$\iff$	$((A \implies B)$	$\wedge$	$(B \implies A)))$
F	F	T	T	T	T	T
F	T	F	T	T	F	F
T	F	F	?	?	?	?
T	T	?	?	?	?	?

(A) F

(B) T

Complex Truth Tables

We may evaluate when a WFF is true by analyzing the truth values of every subexpression

Example (Biconditional Exchange)

A	B	$((A \iff B)$	$\iff$	$((A \implies B)$	$\wedge$	$(B \implies A)))$
F	F	T	T	T	T	T
F	T	F	T	T	F	F
T	F	F	?	F	?	?
T	T	?	?	?	?	?

(A) F

(B) T

Complex Truth Tables

We may evaluate when a WFF is true by analyzing the truth values of every subexpression

Example (Biconditional Exchange)

A	B	$((A \iff B)$	$\iff$	$((A \implies B)$	$\wedge$	$(B \implies A)))$
F	F	T	T	T	T	T
F	T	F	T	T	F	F
T	F	F	?	F	?	T
T	T	?	?	?	?	?

(A) F

(B) T

Complex Truth Tables

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Example (Biconditional Exchange)

A	B	$((A \iff B)$	$\iff$	$((A \implies B)$	$\wedge$	$(B \implies A)))$
F	F	T	T	T	T	T
F	T	F	T	T	F	F
T	F	F	?	F	F	T
T	T	?	?	?	?	?

(A) F

(B) T

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Example (Biconditional Exchange)

A	B	$((A \iff B)$	$\iff$	$((A \implies B)$	$\wedge$	$(B \implies A)))$
F	F	T	T	T	T	T
F	T	F	T	T	F	F
T	F	F	T	F	F	T
T	T	?	?	?	?	?

(A) F

(B) T

Complex Truth Tables

We may evaluate when a WFF is true by analyzing the truth values of every subexpression

Example (Biconditional Exchange)

A	B	$((A \iff B)$	$\iff$	$((A \implies B)$	$\wedge$	$(B \implies A)))$
F	F	T	T	T	T	T
F	T	F	T	T	F	F
T	F	F	T	F	F	T
T	T	T	?	?	?	?

(A) F

(B) T

Complex Truth Tables

We may evaluate when a WFF is true by analyzing the truth values of every subexpression

Example (Biconditional Exchange)

A	B	$((A$	$\iff$	$B)$	$\iff$	$((A$	$\implies$	$B)$	$\land$	$(B$	$\implies$	$A)))$
F	F		T		T		T		T		T	
F	T		F		T		T		F		F	
T	F		F		T		F		F		T	
T	T		T		?		T		?		?	

(A) F

(B) T

Complex Truth Tables

We may evaluate when a WFF is true by analyzing the truth values of every subexpression

Example (Biconditional Exchange)

A	B	$((A \iff B)$	$\iff$	$((A \implies B)$	$\wedge$	$(B \implies A)))$
F	F	T	T	T	T	T
F	T	F	T	T	F	F
T	F	F	T	F	F	T
T	T	T	?	T	?	T

(A) F

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F	T	F	T	T	F	F
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F	F	T	T	T	T	T
F	T	F	T	T	F	F
T	F	F	T	F	F	T
T	T	T	T	T	T	T

Types of Propositions

Definition

A tautology is a proposition that is always true.

Definition

A contradiction is a proposition that is always false.

Definition

A contingency is a proposition that is neither a tautology nor a contradiction (i.e., is sometimes true and sometimes false).

Multiple Choice Question

$$((A \iff B) \iff ((A \implies B) \land (B \implies A)))$$

Why was Biconditional Exchange a tautology?

(A) When A and B are particular propositions, it happens to work

(B) No matter what WFFs A and B are, the equivalence holds by its very form

(C) We provided an airtight argument via truth table

(D) It's not a tautology

Multiple Choice Question

P	$(P$	$\wedge$	$\neg P)$
F		F	T
T		F	F

What is $(P \wedge \neg P)$?

(A) A tautology

(B) A contradiction

(C) A contingency

Multiple Choice Question

P	$(P$	$\vee$	$\neg P)$
F		T	T
T		T	F

What is $(P \vee \neg P)$?

(A) A tautology

(B) A contradiction

(C) A contingency

Multiple Choice Question

P	Q	$(\neg Q$	$\implies$	$\neg P)$
F	F	T	T	T
F	T	F	T	T
T	F	T	F	F
T	T	F	T	F

What is $(\neg Q \implies \neg P)$?

(A) A tautology

(B) A contradiction

(C) A contingency

Interesting Tautologies

Definition

Two WFFs, A and B, are logically equivalent iff $(A \iff B)$ is a tautology

Example (Contrapositive)

The contrapositive of an implication $(P \implies Q)$ is

$$(\neg Q \implies \neg P)$$

and it is logically equivalent to the original implication; i.e.,

$$((P \implies Q) \iff (\neg Q \implies \neg P))$$

Multiple Choice Question

How would we know that an implication is logically equivalent to its contrapositive?

(A) By writing a truth table

(B) By intuition

(C) By showing examples

(D) By guessing

Multiple Choice Question

What is the contrapositive of the following formula?

$$(a \implies (b \vee c))$$

(A) $(\neg a \implies \neg(b \vee c))$
(B) $((b \vee c) \implies a)$
(C) $(\neg(b \vee c) \implies \neg a)$
(D) $\neg(a \implies (b \vee c))$

Contrapositive vs Converse vs Negation

Note

Given the implication $(P \implies Q)$, do not confuse the following:

The Contrapositive: $(\neg Q \implies \neg P)$

The Converse: $(Q \implies P)$ (sometimes denoted $(P \impliedby Q)$)

The Negation: $\neg(P \implies Q)$

They are (pairwise) nonequivalent!

Multiple Choice Question

Which one of the following is equivalent to $(\neg P \implies \neg Q)$?

(A) $(P \implies Q)$

(B) $\neg(P \implies Q)$

(C) $(Q \implies P)$

(D) $\neg(Q \implies P)$

Useful Equivalences

Definition (De Morgan's Laws)

$$(\neg(A \wedge B) \iff (\neg A \vee \neg B))$$
$$(\neg(A \vee B) \iff (\neg A \wedge \neg B))$$

Definition (Operators In Terms of Implication & Negation)

$$((A \wedge B) \iff \neg(A \implies \neg B))$$
$$((A \vee B) \iff (\neg A \implies B))$$
$$((A \iff B) \iff \neg((A \implies B) \implies \neg(B \implies A)))$$

Propositional Logic

Definition

Logic is a systematic way of thinking that allows us to deduce new information from old information

Example

Suppose the following two propositions are true:

- Circle X has a radius of 3 units
- If a circle has a radius of r units, then it has an area of πr^2 square units

Therefore, we can say (logically) that Circle X has an area of 9π square units

Note

Logic is about deducing information correctly, not necessarily deducing correct information (e.g., if Circle X actually had a different radius)

Multiple Choice Question

For the sake of argument, suppose the following proposition is true:

- If the glove doesn't fit, you must acquit

What logically follows from this truth?

(A) You must acquit

(B) You must not acquit

(C) You both must acquit and must not acquit

(D) You can't say for certain whether you must or must not acquit

Multiple Choice Question

For the sake of argument, suppose the following propositions are both true:

- If the glove doesn't fit, you must acquit
- The glove doesn't fit

What logically follows from these truths?

(A) You must acquit

(B) You must not acquit

(C) You both must acquit and must not acquit

(D) You can't say for certain whether you must or must not acquit

Analysis of Arguments

Definition

An argument is a list of premises which (taken all together) supposedly imply a conclusion.

P_1
P_2
P_3
$\vdots$
P_n

$\therefore Q$

We say an argument is valid iff

$$((P_1 \land P_2 \land P_3 \land \ldots \land P_n) \implies Q)$$

is a tautology.

Multiple Choice Question

Is the following argument logically valid?

If today is Sunday, then tomorrow is Monday.
Today is not Sunday.

$\therefore$ Tomorrow is not Monday.

(A) Valid

(B) Invalid

Multiple Choice Question

Is the following argument logically valid?

If it rains, then I wear a hat.
It never rains.

$\therefore$ I never wear a hat.

(A) Valid

(B) Invalid

Multiple Choice Question

Is the following argument logically valid?

$$P \implies Q$$
$$\neg P$$

$$\therefore \neg Q$$

(A) Valid

(B) Invalid