# Sets

## CS 130

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# Sets

### Definition

A set is an unordered collection of objects in which no object appears twice.
A set can be defined by enumerating its elements as a comma-separated list enclosed with curly braces.

### Examples

- $\{0, 1\}$ (a set of two elements, 0 and 1, in no particular order)
- $\{1, 0\}$ (a set of two elements, 0 and 1, in no particular order)
- $\{\text{red}, \text{green}, \text{blue}\}$ (a set of three elements—colors)
- $\{\text{Ronnie James Dio}\}$ (a set of one element—a person)
- $\{\text{Ronnie}, \text{James}, \text{Dio}\}$ (a set of three names)

# Multiple Choice Questions

1. Can a set have zero elements?
   (A) Yes
   (B) No
2. Can a set contain other sets?
   (A) Yes
   (B) No
3. Can a set have an infinite number of elements?
   (A) Yes
   (B) No

# The Empty Set

**Definition**

The empty set is the set with no elements in it, $\{\ \}$

**Danger, Will Robinson!**

You'll often see $\{\ \}$ denoted $\varnothing$

- That is, $\{\ \} = \varnothing$
- To avoid confusion, I'll always use the explicit notation $\{\ \}$
- I can't promise the same thing of your textbook

So, just to be clear...

# Multiple Choice Question

Is the set

$$\{\varnothing\}$$

the same thing as the empty set?

(A) No

(B) Do not want

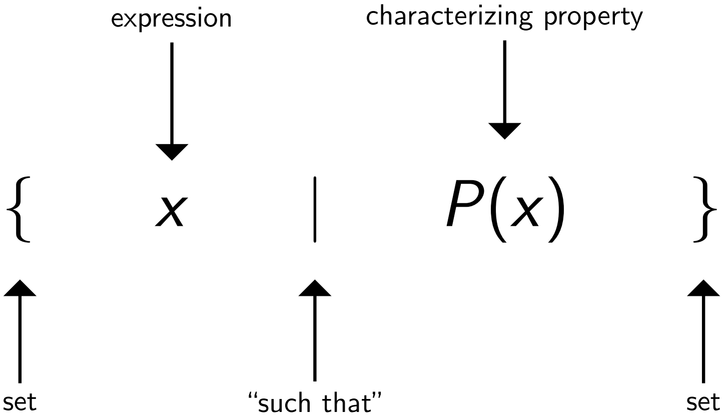
(C) Absolutely not

(D) Nooooooooo...

# Set-Builder Notation

**Definition**

A set can be defined using a characterizing property in set-builder notation.




$$\{ x \mid P(x) \}$$

# Multiple Choice Question

How many elements are in the following set, $S$?

$$S = \{x \mid x \text{ is an integer} \quad \wedge \quad 3 < x \leq 7\}$$

(A) 0
(B) 4
(C) 5
(D) Infinitely many

# Multiple Choice Question

What is another way of writing the following set, $S$?

$$S = \{8, 6, 7, 5, 3, 0, 9\}$$

(A) $S = \{x \mid x \text{ is a natural number} \quad \wedge \quad x < 10\}$

(B) $S = \{x \mid x = 8 \wedge x = 6 \wedge x = 7 \wedge x = 5 \wedge x = 3 \wedge x = 0 \wedge x = 9\}$

(C) $S = \{\left|\sqrt{x}\right| \mid x = 0 \vee x = 9 \vee x = 25 \vee x = 36 \vee x = 49 \vee x = 64 \vee x = 81\}$

(D) $S = \{x^2 \vee x \mid x = 0 \vee x = 3 \vee x = 5 \vee x = 6 \vee x = 7 \vee x = 8\}$

# Multiple Choice Question

What is another way of writing the following set, $S$?

$$S = \{x^2 \mid x \text{ is a natural number} \quad \wedge \quad x < 5\}$$

(A) $S = \{0^2, 1^2, 2^2, 3^2, 4^2\}$

(B) $S = \{0, 1, 4, 9, 16\}$

(C) $S = \{x^2 \mid x \text{ is an integer} \quad \wedge \quad 0 \leq x < 5\}$

(D) All of the above

# Basic Notation

## Definition (Membership Notation)

$a \in A$ (object $a$ is a member (or an element) of set $A$)

$a \notin A$ (object $a$ is not a member of set $A$)

## Definitions (Set Comparisons)

For arbitrary sets $A$ and $B$, we can make the following comparisons:

$$A = B \iff \forall x[x \in A \iff x \in B] \quad \text{(equal)}$$
$$A \subseteq B \iff \forall x[x \in A \implies x \in B] \quad \text{(subset)}$$
$$A \supseteq B \iff \forall x[x \in B \implies x \in A] \quad \text{(superset)}$$
$$A \subset B \iff A \subseteq B \wedge A \neq B \quad \text{(proper subset)}$$
$$A \supset B \iff A \supseteq B \wedge A \neq B \quad \text{(proper superset)}$$

# Multiple Choice Question

Which of the following correctly defines set-builder notation?

(A) $S = \{x \mid P(x)\} \iff \forall x[P(x) \iff x \in S]$

(B) $S = \{x \mid P(x)\} \iff \forall x[P(x) \implies x \in S]$

(C) $S = \{x \mid P(x)\} \iff \forall x[P(x) \impliedby x \in S]$

(D) $S = \{x \mid P(x)\} \iff \forall x[\neg P(x) \implies x \notin S]$

# Multiple Choice Question

Let

$$A = \{x \mid x \text{ is a natural number} \quad \wedge \quad x \geq 5\}$$
$$B = \{10, 12, 16, 20\}$$
$$C = \{x \mid \exists y[y \text{ is a natural number} \quad \wedge \quad x = 2y]\}$$

Is the following true or false?

$$B \subseteq C$$

(A) True

(B) False

# Multiple Choice Question

Let

$$A = \{x \mid x \text{ is a natural number} \quad \wedge \quad x \geq 5\}$$
$$B = \{10, 12, 16, 20\}$$
$$C = \{x \mid \exists y[y \text{ is a natural number} \quad \wedge \quad x = 2y]\}$$

Is the following true or false?

$$B \subset A$$

(A) True

(B) False

# Multiple Choice Question

Let

$$A = \{x \mid x \text{ is a natural number} \quad \wedge \quad x \geq 5\}$$
$$B = \{10, 12, 16, 20\}$$
$$C = \{x \mid \exists y[y \text{ is a natural number} \quad \wedge \quad x = 2y]\}$$

Is the following true or false?

$$A \subseteq C$$

(A) True

(B) False

# Multiple Choice Question

Let

$$A = \{x \mid x \text{ is a natural number} \quad \wedge \quad x \geq 5\}$$
$$B = \{10, 12, 16, 20\}$$
$$C = \{x \mid \exists y[y \text{ is a natural number} \quad \wedge \quad x = 2y]\}$$

Is the following true or false?

$$26 \in C$$

(A) True

(B) False

# Multiple Choice Question

Let

$$A = \{x \mid x \text{ is a natural number} \quad \wedge \quad x \geq 5\}$$
$$B = \{10, 12, 16, 20\}$$
$$C = \{x \mid \exists y[y \text{ is a natural number} \quad \wedge \quad x = 2y]\}$$

Is the following true or false?

$$\{11, 12, 13\} \subseteq A$$

(A) True

(B) False

# Multiple Choice Question

Let

$$
\begin{aligned}
A &= \{x \mid x \text{ is a natural number} \quad \wedge \quad x \geq 5\} \\
B &= \{10, 12, 16, 20\} \\
C &= \{x \mid \exists y[y \text{ is a natural number} \quad \wedge \quad x = 2y]\}
\end{aligned}
$$

Is the following true or false?

$$\{11, 12, 13\} \subset C$$

(A) True

(B) False

# Multiple Choice Question

Let

$$A = \{x \mid x \text{ is a natural number} \quad \wedge \quad x \geq 5\}$$
$$B = \{10, 12, 16, 20\}$$
$$C = \{x \mid \exists y[y \text{ is a natural number} \quad \wedge \quad x = 2y]\}$$

Is the following true or false?

$$\{12\} \in B$$

(A) True

(B) False

# Multiple Choice Question

Let

$$A = \{x \mid x \text{ is a natural number} \quad \wedge \quad x \geq 5\}$$
$$B = \{10, 12, 16, 20\}$$
$$C = \{x \mid \exists y[y \text{ is a natural number} \quad \wedge \quad x = 2y]\}$$

Is the following true or false?

$$\{12\} \subseteq B$$

(A) True

(B) False

# Multiple Choice Question

Let

$$A = \{x \mid x \text{ is a natural number} \quad \wedge \quad x \geq 5\}$$
$$B = \{10, 12, 16, 20\}$$
$$C = \{x \mid \exists y[y \text{ is a natural number} \quad \wedge \quad x = 2y]\}$$

Is the following true or false?

$$\{x \mid x \text{ is a natural number} \wedge x < 20\} \not\subseteq B$$

(A) True

(B) False

# Multiple Choice Question

Let

$$
\begin{aligned}
A &= \{x \mid x \text{ is a natural number} \quad \wedge \quad x \geq 5\} \\
B &= \{10, 12, 16, 20\} \\
C &= \{x \mid \exists y[y \text{ is a natural number} \quad \wedge \quad x = 2y]\}
\end{aligned}
$$

Is the following true or false?

$$5 \subseteq A$$

(A) True
(B) False

# Multiple Choice Question

Let

$$A = \{x \mid x \text{ is a natural number} \quad \wedge \quad x \geq 5\}$$
$$B = \{10, 12, 16, 20\}$$
$$C = \{x \mid \exists y[y \text{ is a natural number} \quad \wedge \quad x = 2y]\}$$

Is the following true or false?

$$\{\{\}\} \subseteq B$$

(A) True

(B) False

# Multiple Choice Question

Let

$$A = \{x \mid x \text{ is a natural number} \quad \wedge \quad x \geq 5\}$$
$$B = \{10, 12, 16, 20\}$$
$$C = \{x \mid \exists y[y \text{ is a natural number} \quad \wedge \quad x = 2y]\}$$

Is the following true or false?

$$\{\ \} \notin A$$

(A) True

(B) False

# Multiple Choice Question

Let

$$A = \{x \mid x \text{ is a natural number} \quad \wedge \quad x \geq 5\}$$
$$B = \{10, 12, 16, 20\}$$
$$C = \{x \mid \exists y[y \text{ is a natural number} \quad \wedge \quad x = 2y]\}$$

Is the following true or false?

$$\{\ \} \subseteq A$$

(A) True

(B) False

# Multiple Choice Question

Let

$$A = \{x \mid x \text{ is a natural number} \quad \wedge \quad x \geq 5\}$$
$$B = \{10, 12, 16, 20\}$$
$$C = \{x \mid \exists y[y \text{ is a natural number} \quad \wedge \quad x = 2y]\}$$

Is the following true or false?

$$\{\ \} \subseteq B$$

(A) True

(B) False

# Multiple Choice Question

Let

$$A = \{x \mid x \text{ is a natural number} \quad \wedge \quad x \geq 5\}$$
$$B = \{10, 12, 16, 20\}$$
$$C = \{x \mid \exists y[y \text{ is a natural number} \quad \wedge \quad x = 2y]\}$$

Is the following true or false?

$$\{\ \} \subseteq C$$

(A) True

(B) False

# Multiple Choice Question

Let $A = \{\text{red}, \text{green}, \text{blue}\}$.
How many different subsets does $A$ have?

(A) 3

(B) 6

(C) 8

(D) None of the above

# Multiple Choice Question

Let $A$ be a set with $n$ different elements.
How many different subsets does $A$ have?

(A) $n$

(B) $2n$

(C) $2(n+1)$

(D) $2^n$

# Cardinality & Power Sets

**Definition**

The cardinality of a set $A$ is the number of elements within $A$.
Denoted $|A|$ (or sometimes $\|A\|$).

**Definition**

Let $A$ be an arbitrary set. The power set of $A$, denoted $\wp(A)$, is the set of all subsets of $A$.
That is,

$$\wp(A) = \{B \mid B \subseteq A\}$$

# Multiple Choice Question

What is $|\wp(A)|$ for any arbitrary set, $A$?

(A) $2\,|A|$

(B) $2^{|A|}$

(C) $|A|^2$

(D) None of the above

# Complement of a Set

**Definition**

Conceptually, any set $A$ can be considered a subset of a universal set, $U$. That is, $A \in \wp(U)$.

The complement of a set $A$ is the set of all things not included in $A$. In set-builder notation,

$$\overline{A} = \{x \mid x \in U \land x \notin A\}$$

# Multiple Choice Question

Suppose $U = \{\text{red}, \text{yellow}, \text{green}, \text{blue}\}$.

Let $A = \{\text{red}, \text{green}\}$.

What is $\overline{A}$?

(A) $\{\}$

(B) $\{\text{yellow}, \text{blue}\}$

(C) $\{\{\}, \{\text{yellow}\}, \{\text{blue}\}\}$

(D) $\{\text{red}, \text{yellow}, \text{green}, \text{blue}\}$

# Common Notation

**Definitions (Well-Known Sets)**

$$
\begin{aligned}
\mathbb{N} &= \{x \mid x \text{ is a natural number}\} \\
&= \{0, 1, 2, \ldots\} && \text{(sometimes without 0)} \\
\mathbb{Z} &= \{x \mid x \text{ is an integer}\} \\
&= \{\ldots, -2, -1, 0, 1, 2, \ldots\} \\
\mathbb{Q} &= \{x \mid x \text{ is a rational number}\} \\
\mathbb{R} &= \{x \mid x \text{ is a real number}\} \\
\mathbb{C} &= \{x \mid x \text{ is a complex number}\}
\end{aligned}
$$

**Note**

To save space, you often see notation like $\forall x \in \mathbb{N}[\ldots]$ instead of $\forall x[x \in \mathbb{N} \implies \ldots]$

# Multiple Choice Question

$$\mathbb{Q} = \{x \mid x \text{ is a rational number}\}$$

Which of the following is an equivalent way of writing $\mathbb{Q}$?

(A) $\mathbb{Q} = \{x/y \mid x \in \mathbb{N} \land y \in \mathbb{N}\}$

(B) $\mathbb{Q} = \{x/y \mid x \in \mathbb{Z} \land y \in \mathbb{Z}\}$

(C) $\mathbb{Q} = \{x/y \mid x \in \mathbb{Z} \land y \in \mathbb{Z} \land y \neq 0\}$

(D) None of the above

# Multiple Choice Question

Let

$$A = \{x \mid x \in \mathbb{N} \land \exists y \in \mathbb{N}[x = 8y]\}$$
$$B = \{x \mid x \in \mathbb{N} \land \exists y \in \mathbb{N}[x = 4y]\}$$

Proof ($A \subseteq B$).

. . . □

How do we prove $A \subseteq B$?

(A) Let $x$ be an arbitrary element of $B$; show $x$ must be in $A$

(B) Show that $A = B$

(C) Show that $B \not\subseteq A$

(D) None of the above

# Multiple Choice Question

Let

$$A = \{x \mid x \in \mathbb{N} \land \exists y \in \mathbb{N}[x = 8y]\}$$
$$B = \{x \mid x \in \mathbb{N} \land \exists y \in \mathbb{N}[x = 4y]\}$$

Proof ($A \subseteq B$).

Consider an arbitrary $x \in A$.
By the definition of $A$, $x \in \mathbb{N}$ and there is some constant $y \in \mathbb{N}$ such that $x = 8y$.
Thus. . . □

Where should we go next?

(A) Conclude that $x \in B$

(B) Show that $x = 4y$

(C) Show that $\exists y[x = 4y]$

(D) Show that $x \in \mathbb{N}$

# Multiple Choice Question

Let

$$A = \{x \mid x \in \mathbb{N} \land \exists y \in \mathbb{N}[x = 8y]\}$$
$$B = \{x \mid x \in \mathbb{N} \land \exists y \in \mathbb{N}[x = 4y]\}$$

Proof ($A \subseteq B$).

Consider an arbitrary $x \in A$.
By the definition of $A$, $x \in \mathbb{N}$ and there is some constant $y \in \mathbb{N}$ such that $x = 8y$.
Thus, $x = 4 \cdot 2 \cdot y$, so $x$ is also a multiple of 4 (because $2y \in \mathbb{N}$).
Therefore, ... □

Where should we go next?

(A) Conclude that $x \in B$
(B) Conclude that $x = 4y$
(C) Conclude that $\exists y[x = 4y]$
(D) Conclude that $x \subseteq B$

# Proofs Involving Sets

Let

$$A = \{x \mid x \in \mathbb{N} \land \exists y \in \mathbb{N}[x = 8y]\}$$
$$B = \{x \mid x \in \mathbb{N} \land \exists y \in \mathbb{N}[x = 4y]\}$$

**Proof ($A \subseteq B$).**

Consider an arbitrary $x \in A$.
By the definition of $A$, $x \in \mathbb{N}$ and there is some constant $y \in \mathbb{N}$ such that $x = 8y$.
Thus, $x = 4 \cdot 2 \cdot y$, so $x$ is also a multiple of 4 (because $2y \in \mathbb{N}$).
Therefore, $x \in B$.
Since $x$ was arbitrary, $A \subseteq B$. □

# Multiple Choice Question

Suppose we wanted to prove two sets $A$ and $B$ were equal. I.e.,

$$A = B$$

How should we do that?

(A) Assume $x \in A$, show $x \in B$

(B) Assume $x \in B$, show $x \in A$

(C) Use a series of equivalences

(D) Show both (A) and (B)

# Multiple Choice Question

$$A = \{x \mid x \in \mathbb{N} \land x^2 < 15\} \quad = \quad B = \{x \mid x \in \mathbb{N} \land 2x < 7\}$$

*Proof.*

. . .

□

Which method do you think is going to be easiest here?

(A) Show the $\subseteq$ and $\supseteq$ directions

(B) Use a chain of equivalences

(C) Both of the above

(D) None of the above

# Multiple Choice Question

$$A = \{x \mid x \in \mathbb{N} \land x^2 < 15\} \quad = \quad B = \{x \mid x \in \mathbb{N} \land 2x < 7\}$$

Proof.

($\subseteq$) ...

($\supseteq$) ...

□

How do we show the $A \subseteq B$?

(A) Assume $a \in A$, show $a \in B$

(B) Use a chain of equivalences

(C) Assume $b \in B$, show $b \in A$

(D) Use a known fact from algebra

# Multiple Choice Question

$$A = \{x \mid x \in \mathbb{N} \wedge x^2 < 15\} \quad = \quad B = \{x \mid x \in \mathbb{N} \wedge 2x < 7\}$$

Proof.

($\subseteq$) Consider an arbitrary $a \in A$. ...

($\supseteq$) ...

□

How do we show the $A \supseteq B$?

(A) Assume $a \in A$, show $a \in B$

(B) Use a chain of equivalences

(C) Assume $b \in B$, show $b \in A$

(D) Use a known fact from algebra

# Multiple Choice Question

$$A = \{x \mid x \in \mathbb{N} \wedge x^2 < 15\} \quad = \quad B = \{x \mid x \in \mathbb{N} \wedge 2x < 7\}$$

Proof.

($\subseteq$) Consider an arbitrary $a \in A$. ...

($\supseteq$) Consider an arbitrary $b \in B$. ...

$\square$

What does $a \in A$ give us?

(A) $a \in \mathbb{N}$

(B) $a^2 < 15$

(C) $2a < 7$

(D) Both (A) and (B)

# Multiple Choice Question

$$A = \{x \mid x \in \mathbb{N} \wedge x^2 < 15\} \quad = \quad B = \{x \mid x \in \mathbb{N} \wedge 2x < 7\}$$

Proof.

($\subseteq$) Consider an arbitrary $a \in A$. Then $a \in \mathbb{N}$ and $a^2 < 15$. ...

($\supseteq$) Consider an arbitrary $b \in B$. ...

□

What are the only possible values for $a$?

(A) 0, 1, 2

(B) 0, 1, 2, 3

(C) 0, 1, 2, 3, 4

(D) 0, 1, 2, 3, 4, 5

# Multiple Choice Question

$$A = \{x \mid x \in \mathbb{N} \wedge x^2 < 15\} \quad = \quad B = \{x \mid x \in \mathbb{N} \wedge 2x < 7\}$$

Proof.

($\subseteq$) Consider an arbitrary $a \in A$. Then $a \in \mathbb{N}$ and $a^2 < 15$. Because of these, $a$ could only possibly be a natural number 0–3. ...

($\supseteq$) Consider an arbitrary $b \in B$. ...

□

How do we know $2a < 7$?

(A) $2a < 7$ for all possible values of $a$

(B) We could prove it by induction on $a$

(C) $a^2 < 15 \implies a < \sqrt{15}$

(D) We don't; this isn't what we want to conclude

# Multiple Choice Question

$$A = \{x \mid x \in \mathbb{N} \wedge x^2 < 15\} \quad = \quad B = \{x \mid x \in \mathbb{N} \wedge 2x < 7\}$$

Proof.

($\subseteq$) Consider an arbitrary $a \in A$. Then $a \in \mathbb{N}$ and $a^2 < 15$. Because of these, $a$ could only possibly be a natural number 0–3. The double of any of these numbers is less than 7, so $a \in B$.

($\supseteq$) Consider an arbitrary $b \in B$. ...

□

What does $b \in B$ give us?

(A) $a \in B$

(B) $2a < 7$

(C) $2b < 7$

(D) None of the above

# A Convoluted Example

$$A = \{x \mid x \in \mathbb{N} \land x^2 < 15\} \quad = \quad B = \{x \mid x \in \mathbb{N} \land 2x < 7\}$$

Proof.

($\subseteq$) Consider an arbitrary $a \in A$. Then $a \in \mathbb{N}$ and $a^2 < 15$. Because of these, $a$ could only possibly be a natural number 0–3. The double of any of these numbers is less than 7, so $a \in B$.

($\supseteq$) Consider an arbitrary $b \in B$. Then $b \in \mathbb{N}$ and $2b < 7$. Because of these, $b$ could only possibly be a natural number 0–3. The square of any of these numbers is less than 15, so $b \in A$.

□

# Multiple Choice Question

How does the same "two direction" strategy apply to proving something of the following form?

$$A \iff B$$

(A) Proving $A \iff B$ shows that $A = B$

(B) To prove $A \iff B$, first show $A \implies B$, then show $B \implies A$

(C) To prove $A \iff B$, first show $A \subseteq B$, then show $B \subseteq A$

(D) None of the above

# Multiple Choice Question

We needn't necessarily use the "two direction" style proof. We might be able to just use a series of equivalences.

Proof ($A = B \iff (A \subseteq B) \wedge (B \subseteq A)$).

$$A = B \iff \ldots \qquad \text{(defn =)}$$

□

Instead of having the $\implies$ / $\impliedby$ cases, we could replace $A = B$ with its definition.

What was the definition of $A = B$?

(A) $\forall x[A \subseteq B \wedge B \subseteq A]$

(B) $\forall x[x \in A \implies x \in B]$

(C) $\forall x[x \in A \impliedby x \in B]$

(D) $\forall x[x \in A \iff x \in B]$

# Multiple Choice Question

We needn't necessarily use the "two direction" style proof. We might be able to just use a series of equivalences.

Proof $(A = B \iff (A \subseteq B) \wedge (B \subseteq A))$.

$$\begin{aligned} A = B &\iff \forall x[x \in A \iff x \in B] && \text{(defn =)} \\ &\iff \ldots && \text{("Biconditional Exchange")} \end{aligned}$$

□

How can we change the " $\iff$ " inside the $\forall x[\ldots]$?

(A) $\forall x[(A \subseteq B) \implies (B \subseteq A)]$

(B) $\forall x[(x \in A \implies x \in B) \wedge (x \in B \implies x \in A)]$

(C) $\forall x[(x \in A \implies x \in B) \wedge (x \in B \impliedby x \in A)]$

(D) None of the above

# Multiple Choice Question

Proof $(A = B \iff (A \subseteq B) \wedge (B \subseteq A))$.

$$
\begin{aligned}
A = B &\iff \forall x[x \in A \iff x \in B] && \text{(defn } =) \\
&\iff \forall x[(x \in A \implies x \in B) \wedge (x \in B \implies x \in A)] \\
& && \text{(“Biconditional Exchange”)} \\
&\iff \forall x[x \in A \implies x \in B] \wedge \forall x[x \in B \implies x \in A] \\
& && \text{(unproven result ☹)} \\
&\iff \dots && \text{(defn } \subseteq, \supseteq)
\end{aligned}
$$

□

What can we conclude?

(A) $(A \subseteq B) \wedge (B \subseteq A)$
(B) $(A \subseteq B) \wedge (B \supseteq A)$
(C) $(A \supseteq B) \wedge (B \subseteq A)$
(D) $(A \supseteq B) \wedge (B \supseteq A)$

# Another Set Proof

Proof $(A = B \iff (A \subseteq B) \land (B \subseteq A))$.

$$
\begin{aligned}
& A = B \\
\iff & \forall x[x \in A \iff x \in B] \\
\iff & \forall x[(x \in A \implies x \in B) \land (x \in B \implies x \in A)] \\
\iff & \forall x[x \in A \implies x \in B] \land \forall x[x \in B \implies x \in A] \\
\iff & (A \subseteq B) \land (B \subseteq A)
\end{aligned}
$$

□

# Set Operations

**Definitions**

From two arbitrary sets $A$ and $B$, we can form any of the following new sets.

$$\begin{aligned} A \cup B &= \{x \mid x \in A \vee x \in B\} && \text{(union)} \\ A \cap B &= \{x \mid x \in A \wedge x \in B\} && \text{(intersection)} \\ A \setminus B &= \{x \mid x \in A \wedge x \notin B\} && \text{(set difference)} \\ A \,\Delta\, B &= (A \setminus B) \cup (B \setminus A) && \text{(symmetric difference)} \end{aligned}$$

# Multiple Choice Question

Let the universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Furthermore, let

$$A = \{1, 2, 3, 5, 10\}$$
$$B = \{2, 4, 7, 8, 9\}$$
$$C = \{5, 8, 10\}$$

What is the value of the following?

$$A \cup B$$

(A) $\{1, 2, 3, 5, 10, 2, 4, 7, 8, 9\}$
(B) $\{2\}$
(C) $\{1, 2, 3, 4, 5, 7, 8, 9, 10\}$
(D) $\{1, 3, 4, 5, 7, 8, 9, 10\}$

# Multiple Choice Question

Let the universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Furthermore, let

$$A = \{1, 2, 3, 5, 10\}$$
$$B = \{2, 4, 7, 8, 9\}$$
$$C = \{5, 8, 10\}$$

What is the value of the following?

$$A \cap B$$

(A) $\{1, 2, 3, 5, 10, 2, 4, 7, 8, 9\}$
(B) $\{2\}$
(C) $\{1, 2, 3, 4, 5, 7, 8, 9, 10\}$
(D) $\{1, 3, 4, 5, 7, 8, 9, 10\}$

# Multiple Choice Question

Let the universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Furthermore, let

$$A = \{1, 2, 3, 5, 10\}$$
$$B = \{2, 4, 7, 8, 9\}$$
$$C = \{5, 8, 10\}$$

What is the value of the following?

$$A \,\Delta\, B$$

(A) $\{1, 2, 3, 5, 10, 2, 4, 7, 8, 9\}$

(B) $\{2\}$

(C) $\{1, 2, 3, 4, 5, 7, 8, 9, 10\}$

(D) $\{1, 3, 4, 5, 7, 8, 9, 10\}$

# Multiple Choice Question

Let the universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Furthermore, let

$$A = \{1, 2, 3, 5, 10\}$$
$$B = \{2, 4, 7, 8, 9\}$$
$$C = \{5, 8, 10\}$$

What is the value of the following?

$$A \setminus C$$

(A) $\{1, 2, 3, 8, 10\}$
(B) $\{1, 2, 3\}$
(C) $\{1, 2, 3, 8\}$
(D) None of the above

# Multiple Choice Question

Let the universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Furthermore, let

$$A = \{1, 2, 3, 5, 10\}$$
$$B = \{2, 4, 7, 8, 9\}$$
$$C = \{5, 8, 10\}$$

What is the value of the following?

$$\overline{B} \cap (A \cup C)$$

(A) $\{1, 3, 5, 6, 10\}$
(B) $\{1, 2, 3, 5, 8, 10\}$
(C) $\{1, 3, 5, 10\}$
(D) None of the above

# Tuples & Cross Products

### Definition

An *n-tuple* is an ordered sequence of $n$ objects. $n$-tuples are written by listing the $n$ objects within parentheses separated by commas.

### Definition

The cross product (or Cartesian product) of two sets, $A$ and $B$, is the set of 2-tuples:

$$A \times B = \{(x, y) \mid x \in A \wedge y \in B\}$$