Propositional Logic Theorems

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The following are some useful inference rules that may be used as lemmas in any propositional logic proof *after* Homework 2 (unless otherwise noted). It's a good exercise to prove each of these.

	$\vdash$	$(A \implies A)$	(Principle of Identity)
$(A \implies B)$	$\vdash$	$(\neg B \implies \neg A)$	(Contraposition)
$(A \implies B),\quad (B \implies C)$	$\vdash$	$(A \implies C)$	(Syllogism)
$\neg\neg A$	$\vdash$	A	(Double Negation)
A	$\vdash$	$\neg\neg A$	(Double Negation)
$(A \implies B),\quad (A \implies (B \implies C))$	$\vdash$	$(A \implies C)$	(Modus Ponens Deduction)
$\neg B,\quad (A \implies B)$	$\vdash$	$\neg A$	(Modus [Tollendo] Tollens)
$A,\quad \neg A$	$\vdash$	B	(Principle of Explosion)
	$\vdash$	$((\neg A \implies A) \implies A)$	(Law of Clavius)
$A,\quad B$	$\vdash$	$(A \wedge B)$	($\wedge$-introduction)
$(A \wedge B)$	$\vdash$	B	($\wedge$-elimination$_L$)
$(A \wedge B)$	$\vdash$	A	($\wedge$-elimination$_R$)
B	$\vdash$	$(A \vee B)$	($\vee$-introduction$_L$)
A	$\vdash$	$(A \vee B)$	($\vee$-introduction$_R$)
$(A \vee B),\quad \neg A$	$\vdash$	B	($\vee$-elimination$_L$)
$(A \vee B),\quad \neg B$	$\vdash$	A	($\vee$-elimination$_R$)
$(A \iff B)$	$\vdash$	$(A \implies B)$	($\iff$-simplification forward)
$(A \iff B)$	$\vdash$	$(B \implies A)$	($\iff$-simplification backward)
$(A \implies B),\quad (B \implies A)$	$\vdash$	$(A \iff B)$	(Biconditional Exchange Deduction)
	$\vdash$	$((A \implies B) \iff (\neg A \vee B))$	(Material Implication)
	$\vdash$	$((A \wedge B) \iff (B \wedge A))$	($\wedge$-commutativity)
	$\vdash$	$((A \vee B) \iff (B \vee A))$	($\vee$-commutativity)
	$\vdash$	$(A \iff (A \wedge A))$	($\wedge$-idempotence)
	$\vdash$	$(A \iff (A \vee A))$	($\vee$-idempotence)
	$\vdash$	$(\neg(A \wedge B) \iff (\neg A \vee \neg B))$	(De Morgan's Law)
	$\vdash$	$(\neg(A \vee B) \iff (\neg A \wedge \neg B))$	(De Morgan's Law)
	$\vdash$	$((A \implies (B \implies C)) \iff ((A \wedge B) \implies C))$	(Exportation)